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Research article

X-Gamma Lomax Distribution with Different Applications

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Abstract: The X-Gamma Lomax (XGLo) distribution, a new three-parameter modification of the Lomax distribution, was introduced and examined in this study. This distribution's features for reliability and hazard rate are addressed. The methods for estimating the XGLo distribution parameters using maximum likelihood estimation (MLE) and maximum product spacing (MPS) are explained. To compare the MLE and MPS estimate approaches, a numerical investigation is conducted Monte-Carlo simulation. Three real data sets as the cancer data includes failure rates in weeks, 109 days of continuous coal mining occurrences in Great Britain, and remission periods (in months) of a random sample of 128 bladder cancer patients. are used to examine the adaptability and potential of the XGLo distribution. The likelihood ratio test and Kolmogorov- Smirnov test have been used to check the XGLo model is better fits than Lomax model.

Keywords: X-Gamma family; Lomax distribution; likelihood estimation; product spacing; likelihood ratio test.

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1. Introduction

The statistical literature has long discussed the idea of creating new statistical distributions. Pearson's 1895 groundbreaking work, which employed the system of differential equations approach, established

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the standard for creating statistical distributions. After that, many writers used a variety of techniques to create a family of distributions. Lomax in 1954 has researched the Lomax model. It is referred to as the Lomax or Pareto type II distribution and is a crucial distribution for lifetime analysis and business failure data. In addition, it has been widely used in a number of scenarios. In business, economics, and actuarial modelling, the Lomax distribution's probability distribution function (PDF), which has a heavy-tail, is frequently utilized. In order to provide the new distribution more flexibility and possession and to enable it to describe a vast variety of phenomenal data, authors recently produced a number of generalizations for the Lomax distribution.

See Tahir et al. (2015) introduction of the Weibull-Lomax distribution for examples. Rayleigh Lomax distribution was first introduced by Fatima et al. (2018). The chances exponential-Pareto IV distribution was first presented by Baharith et al. (2020), power Lomax distribution with applications has been discussed by Ahmad et al. (2022), Maxwell–Lomax distribution has been introduced by Abiodun and Ishaq (2022), and extended odd Weibull Lomax has been obtained by Alsuhabi et al. (2022.

The Lomax (Lo) distribution has received some attention in more literature it can be used in the reliability engineering discipline and to model a variety of failure characteristics. The cumulative distribution function (CDF) and the probability density function (PDF) of the Lo distribution are respectively as follows

$$F(x;\gamma,\tau) = 1 - \left(1 + \frac{x}{\tau}\right)^{-\gamma};\ x > 0;\ \tau,\gamma > 0, \quad (1.1)$$

$$f(x;\gamma,\tau) = \frac{\gamma}{\tau}\left(1 + \frac{x}{\tau}\right)^{-\gamma-1};x > 0;\ \tau,\gamma > 0. \quad (1.2)$$

Researchers have recently shown a significant deal of interest in the X-Gamma (XG) distribution, which was first described by Sen et al. (2016). By Sen and Chandra, the quasi-XG distribution has been introduced (2017). In contrast to the beta distribution based on the XG distribution, Altun and Hamedani (2018) propose a new bounded distribution using the transformation $Y = e^{-X}$. Sen et al. (2018a) generalization of the XG distribution is based on a unique combination of the exponential and gamma distributions. Sen et al. (2018b) investigated parameter estimation of the XG distribution under the gradually type-II censored sample using various techniques.

Researchers have recently shown a significant deal of interest in the X-Gamma (XG) distribution, which was first described by Sen et al. (2016). By Sen and Chandra (2017), the quasi-XG distribution has been introduced. A new bounded distribution has been introduced by Altun and Hamedani (2018) using the transformation $Y = e^{-X}$ as an alternative to the beta distribution based on the XG distribution. Another generalization of XG distribution has been provided by Sen et al. (2018_a) on the basis of a special mixture of exponential and gamma distributions. Parameter estimation of XG distribution under the progressively type-II censored sample has been studied by Sen et al. (2018_b) by using different methods. Biçer (2019) has investigated the dispersion of the transmuted-XG. Using the transformation $Y = \frac{1}{X}$, Yadav et al. (2019) introduced the inverse X-Gamma distribution. The half-logistic XG distribution was first presented by Bantan et al. (2020) using the half-logistic family. The discrimination study between the Lindley and XG distributions was researched by Sen et al. in 2020.

On the other hand, Cordeiro et al. (2019) have proposed the XG-Generator (XG-G) family to include any distribution into a bigger family. Flexible forms of the XG-G family can be used to model different lifetime data sets. The XG-G family added a parameter with an additional shape parameter of $\alpha > 0$, and its CDF is provided by

$$F(x;\alpha,\psi) = 1 - \frac{[1-G(x;\Phi)]^{\alpha}}{\alpha+1}\left\{1+\alpha-\alpha\ln\big(1-G(x;\Phi)\big)+0.5\alpha^{2}\left[\ln\big(1-G(x;\Phi)\big)\right]^{2}\right\}, \quad (1.3)$$

Where $\alpha > 0$, $G(x;\Phi)$ is a baseline CDF with a parameter vectorψ. The PDF of XG-G family can be expressed as

$$f(x;\alpha,\Phi) = \frac{\alpha}{\alpha+1} g(x;\Phi)[1-G(x;\Phi)]^{\alpha-1}\left\{\alpha+0.5\alpha^{2}\left[\ln\big(1-G(x;\Phi)\big)\right]^{2}\right\} \quad (1.4)$$

where $g(x;\Phi) = dG(x;\Phi)/dx$.

This essay seeks to make two points clear. First, suggest and research the X-Gamma Lomax (XGLo) distribution, a new lifetime distribution based on the XG-G family. The XGLo distribution's reliability and hazard rate features are given. Second, the MLE and MPS methods for estimating the XGLo distribution's parameters are discussed. The performance of the estimators is evaluated through a thorough simulation exercise. Two genuine data sets are used as examples to demonstrate our XGLo model as well as a few other well-known distributions. Compared to certain popular distributions, the XGLo distribution can result in better fits.

The work is structured as follows: Section 2 introduces the description and notation of the XGLo distribution, and Section 3 discusses the distribution's reliability and hazard rate characteristics. We go into XGLo distribution parameter estimate in section 4. Section 5 presents a Monte-Carlo simulation study to contrast the effectiveness of the parameter estimation for various approaches. Three actual data sets' applications are examined in section 6. Finally, we address the findings and conclusions of the present study in section 7.

2. Model Description and Notation

The XGLo distribution has been introduced. The XGLo distribution was created using the XG-G family and Lo distribution. It is represented by the random variable $X \sim XGLo\ (\alpha, \gamma, \tau)$. By using Equations (1.3, 1.4, 1.1 and 1.2), the CDF of XGLo distribution takes this form

$$F(x;\alpha,\gamma,\tau) = 1 - \frac{\left(1+\frac{x}{\tau}\right)^{-\gamma\alpha}}{\alpha+1}\left\{1+\alpha+\gamma\alpha\ln\left(1+\frac{x}{\tau}\right)+0.5\alpha^2\gamma^2\left[\ln\left(1+\frac{x}{\tau}\right)\right]^2\right\}, \quad (2.1)$$

Where $\alpha, \gamma, \tau > 0$ and $x > 0$. The PDF of XGLo distribution is given as:

$$f(x,\alpha,\gamma,\tau) = \frac{\alpha}{\alpha+1}\frac{\gamma}{\tau}\left(1+\frac{x}{\tau}\right)^{-\gamma\alpha-1}\left\{\alpha+0.5\alpha^2\gamma^2\left[\ln\left(1+\frac{x}{\tau}\right)\right]^2\right\} \quad (2.2)$$

Figure 1 display plots of the PDF of the XGLo distribution for some parameters values as follows:

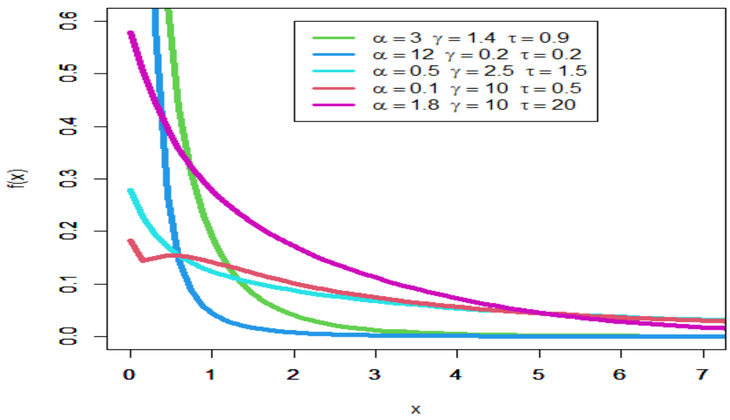

Figure 1. Plots of the PDF of the XGLo distribution for Some Values of Parameters.

3. Reliability Analysis of XGLo Distribution

The XGLo distribution's survival function is provided by

$$S(x;\alpha,\gamma,\tau) = \frac{\left(1+\frac{x}{\tau}\right)^{-\gamma\alpha}}{\alpha+1}\left\{1+\alpha+\gamma\alpha\ln\left(1+\frac{x}{\tau}\right)+0.5\alpha^2\gamma^2\left[\ln\left(1+\frac{x}{\tau}\right)\right]^2\right\} \quad (3.1)$$

The following formula represents the hazard rate function of a lifespan random variable X with an XGLo distribution:

$$h(x;\alpha,\gamma,\tau) = \frac{\alpha\frac{\gamma}{\tau}\left(1+\frac{x}{\tau}\right)^{-1}\left\{\alpha+0.5\alpha^2\gamma^2\left[\ln\left(1+\frac{x}{\tau}\right)\right]^2\right\}}{\left\{1+\alpha+\gamma\alpha\ln\left(1+\frac{x}{\tau}\right)+0.5\alpha^2\gamma^2\left[\ln\left(1+\frac{x}{\tau}\right)\right]^2\right\}} \quad (3.2)$$

The hazard function of the XGLo distribution is plotted in Figure 2 for the following parameter values.

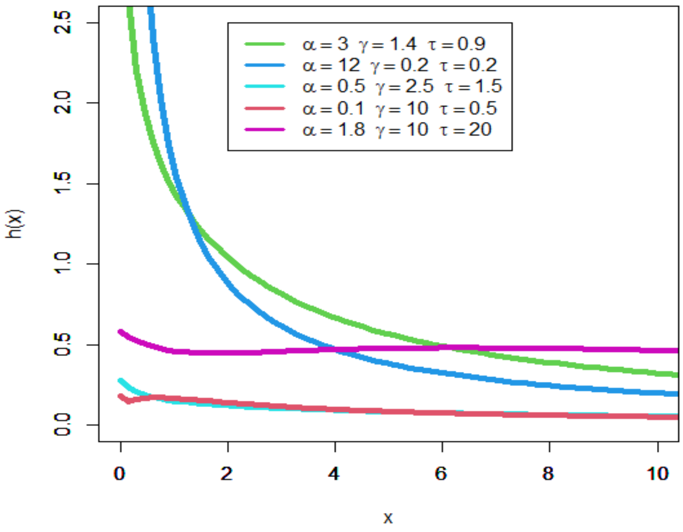

Figure 2. Plots of the hazard of the XGLo with Some Values of the Parameters.

4. Parameter Estimation

This section will go into detail on the parameter estimate of the XGLo distribution utilising the MLE and MPS estimation methods in the presence of the entire sample.

4.1.MLE method

The XGLo distribution's log-likelihood function is given by:

$$l(\alpha,\gamma,\tau) = n\,ln\left(\frac{\alpha}{\alpha+1}\right) + n[\ln(\gamma) - \ln(\tau)] - (\gamma\alpha+1)\sum_{i=1}^{n}\ln\left(1+\frac{x_i}{\tau}\right)$$
$$+\sum_{i=1}^{n} ln\left\{\alpha + 0.5\alpha^2\gamma^2\left[\ln\left(1+\frac{x_i}{\tau}\right)\right]^2\right\}. \tag{4.1}$$

Equation (4.1) can be directly maximised by solving the non-linear likelihood equations produced by differentiating Equation (4.1) with respect to $\vartheta, \alpha, \lambda$, and equating to zero using the R package's optim function. The following are the non-linear likelihood equations:

$$\frac{\partial l(\alpha,\gamma,\tau)}{\partial\alpha} = \frac{n}{\alpha(\alpha+1)} - \gamma\sum_{i=1}^{n}\ln\left(1+\frac{x_i}{\tau}\right) + \sum_{i=1}^{n}\frac{1+\alpha\gamma^2\left[\ln\left(1+\frac{x_i}{\tau}\right)\right]^2}{\alpha+0.5\alpha^2\gamma^2\left[\ln\left(1+\frac{x_i}{\tau}\right)\right]^2},$$

$$\frac{\partial l(\alpha,\gamma,\tau)}{\partial\gamma} = \frac{n}{\gamma} - \alpha\sum_{i=1}^{n}\ln\left(1+\frac{x_i}{\tau}\right) + \sum_{i=1}^{n}\frac{\alpha^2\gamma\left[\ln\left(1+\frac{x_i}{\tau}\right)\right]^2}{\alpha+0.5\alpha^2\gamma^2\left[\ln\left(1+\frac{x_i}{\tau}\right)\right]^2},$$

and

$$\frac{\partial l(\alpha,\gamma,\tau)}{\partial\tau}=\frac{-n}{\tau}-(\gamma\alpha+1)\sum_{i=1}^{n}\frac{-x_i\tau^2}{(x_i+\tau)^2}+\sum_{i=1}^{n}\frac{\alpha^2\gamma^2\ln\left(1+\frac{x_i}{\tau}\right)\frac{-x_i\tau^2}{(x_i+\tau)^2}}{\alpha+0.5\alpha^2\gamma^2\left[\ln\left(1+\frac{x_i}{\tau}\right)\right]^2}.$$

4.2.MPS Method

As an alternative to the MLE approach, the MPS method is used to estimate the parameters of continuous univariate models. According to the XGLo distribution, a random sample $x_1<\cdots<x_n$ of size n with uniform spacings is given by the expression

$$D_i(\alpha,\gamma,\tau)=F(x_i,\alpha,\gamma,\tau)-F(x_{i-1},\alpha,\gamma,\tau);i=1,2,\ldots,n+1$$

where D_i refers to the uniform spacings and $\sum_{i=1}^{n+1}D_i=1$. The MPS estimators can be obtained by maximizing

$$G(\alpha,\gamma,\tau)=\frac{1}{n+1}\sum_{i=1}^{n+1}\ln\big(D_i(\alpha,\gamma,\tau)\big)$$

For more information of MPS method, see Cheng and Amin (1983), Almetwally and Almongy (2019b, a), Almetwally et al. (2019, 2020), El-Sherpieny et al. (2020) and Ahmad and Almetwally (2020).

The MPS of the XGLo distribution's natural logarithm of the product spacing function is given by

$$\ln G(\alpha,\gamma,\tau)=\frac{1}{n+1}\left(\sum_{i=1}^{n+1}\ln\left(\frac{\left(1+\frac{x_{i-1}}{\tau}\right)^{-\gamma\alpha}}{\alpha+1}\left\{1+\alpha+\gamma\alpha\ln\left(1+\frac{x_{i-1}}{\tau}\right)+0.5\alpha^2\gamma^2\left[\ln\left(1+\frac{x_{i-1}}{\tau}\right)\right]^2\right\}\right.\right.$$
$$\left.\left.-\frac{\left(1+\frac{x_i}{\tau}\right)^{-\gamma\alpha}}{\alpha+1}\left\{1+\alpha+\gamma\alpha\ln\left(1+\frac{x_i}{\tau}\right)+0.5\alpha^2\gamma^2\left[\ln\left(1+\frac{x_i}{\tau}\right)\right]^2\right\}\right)\right). \tag{4.2}$$

Since the partial derivatives of the MPS with respect to the unknown parameters cannot be solved explicitly, the MPS of $\alpha,\gamma,$ and τ and can be calculated using numerical techniques such the conjugate-gradients algorithms.

5. Simulation Study

In this section; a Monte Carlo simulation is done to estimate the parameters based on complete sample by using MLE and MPS methods. Using R packages and using the following:

Simulation algorithm: Monte Carlo experiments were carried out based on 5000 random sample for following data generated form XGLo distribution by using numerical analysis, where x_i is distributed as XGLo distribution for different parameters (α,γ,τ) with different actual values of parameter and for different samples sizes $n=30,70,100,150,$ and 200. Equations (4.1 and 4.2) and the R package can be used to determine the parameter estimation. The optimum strategy is one that minimises the estimator's bias and mean squared error (MSE).

The following conclusions can be drawn from Table (1):

1. All of the estimates show the consistency property, which states that the Bias and MSE get smaller as n increases.

2. For the majority of XGLo distribution parameters, the MPS estimates are more efficient relative to MLE.

Table 1: MLE and MPS estimation methods with different values of parameters

$\gamma = 0.5$		τ	0.5				1.5			
			MLE		MPS		MLE		MPS	
α	n		Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
0.4	30	α	0.0683	0.4648	0.1531	0.5315	-0.3091	0.3208	-0.2691	0.0797
		γ	0.4104	0.6662	0.2816	0.3273	0.9014	0.9663	0.6600	0.5437
		τ	1.9918	3.2463	1.5877	3.2328	0.4799	0.5092	0.2870	0.1104
	70	α	0.0396	0.2803	0.0665	0.0880	-0.3012	0.3167	-0.2626	0.0610
		γ	0.3026	0.5430	0.2288	0.2314	0.7612	0.9017	0.5221	0.3678
		τ	1.8065	2.7848	1.4788	2.9588	0.4084	0.5037	0.2548	0.1031
	100	α	0.0135	0.2378	0.0330	0.0574	-0.3023	0.3039	-0.2531	0.0611
		γ	0.3094	0.5304	0.2413	0.2194	0.7081	0.7559	0.4692	0.3561
		τ	1.9203	2.4203	1.3761	2.1804	0.3696	0.4699	0.2462	0.0942
	150	α	0.0014	0.2105	0.0014	0.0429	-0.2933	0.3031	-0.2531	0.0598
		γ	0.3060	0.5160	0.2815	0.2134	0.6123	0.6190	0.4563	0.3352
		τ	1.8616	2.3405	1.2670	1.6698	0.3175	0.3638	0.2375	0.0818
	200	α	-0.0224	0.1933	-0.0192	0.0364	-0.2916	0.2956	-0.2438	0.0419
		γ	0.3362	0.5026	0.3062	0.2039	0.5828	0.4828	0.2010	0.3019
		τ	1.7783	2.2340	0.9639	1.4583	0.3065	0.3655	0.2274	0.0755
1.6	30	α	0.2036	0.5838	0.1197	0.2254	-0.3826	0.5246	-0.3262	0.1796
		γ	0.3002	0.4671	0.1968	0.1154	0.6581	0.7946	0.4290	0.3001
		τ	0.7841	1.0229	0.4494	0.3796	1.5375	1.7263	0.8888	1.2831
	70	α	0.2026	0.4587	0.1012	0.0893	-0.3815	0.4167	-0.3275	0.1603
		γ	0.2175	0.3028	0.1891	0.0638	0.5627	0.6224	0.4163	0.2591
		τ	0.6743	0.7758	0.5061	0.3412	1.4907	1.5574	0.8046	1.2614
	100	α	0.1326	0.3381	0.1007	0.0651	-0.1878	0.3343	-0.2860	0.1388
		γ	0.2158	0.2703	0.1826	0.0500	0.3781	0.4625	0.3864	0.2147
		τ	0.5412	0.6068	0.4433	0.2456	1.4500	1.4674	0.7032	0.9229
	150	α	0.1291	0.2990	0.0873	0.0446	-0.1782	0.3165	-0.2792	0.1346
		γ	0.2031	0.2379	0.1878	0.0448	0.3409	0.4605	0.3710	0.1712
		τ	0.5236	0.5617	0.4572	0.2452	1.2958	1.3396	0.7001	0.9044
	200	α	0.1320	0.2309	0.0788	0.0403	-0.1028	0.3069	-0.2594	0.1189
		γ	0.1993	0.2364	0.1890	0.0435	0.3247	0.3789	0.3727	0.1612
		τ	0.5166	0.5574	0.4516	0.2412	1.0506	1.2570	0.6396	0.8301

6. Application of Real Data Analysis

This section uses three real data sets to examine the adaptability and potential of the XGLo distribution. We offer the Lomax distribution as an application of the XGLo distribution and it sub-model.

Data set I: The cancer data set are given by Lee and Wang (2003) which represent remission times (in months) of a random sample of 128 bladder cancer patients. The data is as follows: “0.08, 2.09, 3.48, 4.87, 6.94 , 8.66, 13.11, 23.63, 0.20, 2.23, 3.52, 4.98, 6.97, 9.02, 13.29, 0.40, 2.26, 3.57, 5.06, 7.09, 9.22, 13.80, 25.74, 0.50, 2.46 , 3.64, 5.09, 7.26, 9.47, 14.24, 25.82, 0.51, 2.54, 3.70, 5.17, 7.28, 9.74, 14.76, 26.31, 0.81, 2.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64, 3.88, 5.32, 7.39, 10.34, 14.83, 34.26, 0.90, 2.69, 4.18, 5.34, 7.59, 10.66, 15.96, 36.66, 1.05, 2.69, 4.23, 5.41, 7.62, 10.75, 16.62, 43.01, 1.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26, 2.83, 4.33, 5.49, 7.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.64, 17.36, 1.40, 3.02, 4.34, 5.71, 7.93, 11.79, 18.10, 1.46, 4.40, 5.85, 8.26, 11.98, 19.13, 1.76, 3.25, 4.50, 6.25, 8.37, 12.02, 2.02, 3.31, 4.51, 6.54, 8.53, 12.03, 20.28, 2.02, 3.36, 6.76, 12.07, 21.73, 2.07, 3.36, 6.93, 8.65, 12.63, 22.69”.

Data set II: The data set, which was used by Nassar et al. (2016), corresponds to the days between 109 consecutive coal-mining incidents in Great Britain. “1, 4, 4, 7, 11, 13, 15, 15, 17, 18, 19, 19, 20, 20, 22, 23, 28, 29, 31, 32, 36, 37, 47, 48, 49, 50, 54, 54, 55, 59, 59, 61, 61, 66, 72, 72, 75, 78, 78, 81, 93, 96, 99, 108, 113, 114, 120, 120, 120, 123, 124, 129, 131, 137, 145, 151, 156, 171, 176, 182, 188, 189, 195, 203, 208, 215, 217, 217, 217, 224, 228, 233, 255, 271, 275, 275, 275, 286, 291, 312, 312, 312, 315, 326, 326, 329, 330, 336, 338, 345, 348, 354, 361, 364, 369, 378, 390, 457, 467, 498, 517, 566, 644, 745, 871, 1312, 1357, 1613, 1630”.

Data set III: All 50 Items Put into Use at t = 0 and Failure Times in Weeks. This data has been introduced by Murthy et al. (2004). The data are “1.578, 1.582, 1.858, 2.595, 2.710, 2.899, 2.940, 3.087, 3.669, 3.848, 4.740, 4.848, 5.170, 5.783, 5.866, 5.872, 6.152, 6.406, 6.839, 7.042, 7.187, 7.262, 7.466, 7.479, 7.481, 8.292, 8.443, 8.475, 8.587, 9.053, 9.172, 9.229, 9.352, 10.046, 11.182, 11.270, 11.490, 11.623, 11.848, 12.695, 14.369, 14.812, 15.662, 16.296, 16.410, 17.181, 17.675, 19.742, 29.022, 29.047”.

Table 2 discussed MLE with stander error (SE), and different measures (AIC, CAIC, BIC, and HQIC) as “Akaike information criterion (AIC), correct Akaike information criterion (CAIC), Bayesian information criterion (BIC) and Hannan–Quinn information criterion (HQIC)”. The Kolmogorov - Smirnov goodness of fit test is employed for real data where we obtained the Kolmogorov- Smirnov distance (KSD) and its Kolmogorov- Smirnov p value (PVKS) indicates that the XGLo and Lomax distribution fits for each data sets in Table 3.

Table 2. MLE, AIC, CAIC, BIC and HQIC for Lomax and XGLo models with different data sets

data			estimates	SE	AIC	CAIC	BIC	HQIC
I	Lo	γ	8.3509	4.7050	831.9923	832.0883	837.6964	834.3099
		τ	69.5592	43.2859				
	XGLo	α	0.2403	0.0129	827.4956	827.6891	836.0516	830.9719
		γ	13.5172	0.0027				
		τ	5.7705	0.0027				
II	Lo	γ	1.7588	0.3779	1412.4198	1412.5330	1417.8025	1414.6027
		τ	237.0444	68.0941				
	XGLo	α	0.7958	0.4972	1406.7917	1407.0203	1414.8658	1410.0660
		γ	5.7299	2.7543				
		τ	314.9754	212.2633				
III	Lo	γ	11.3191	7.0156	329.2707	329.5261	333.0948	330.7270
		τ	97.9268	62.3684				
	XGLo	α	0.2009	0.0171	313.7988	314.3206	319.5349	315.9831
		γ	64.0230	0.0026				
		τ	37.9966	0.0026				

Table 2 shows that the XGLo fits the data better than the Lomax model based on different criteria as the AIC, CAIC, BIC and HQIC values. In order to see how well the XGLo distribution fts this data, the hypotheses are H0: $F = F_{XGLo}$ versus H1: $F \neq F_{XGLo}$. In table 3, the XGLo model has the highest p-value and the lowest distance of KSD value when it compares with Lomax models for different data sets. Furthermore, likelihood ratio test (LRT) has been used to determine the appropriateness of the model. The hypotheses are as follows:

$$\text{H0: } \alpha = 0 \text{ (Lomax) versus H1: } \alpha \neq 0 \text{ (XGLo)}$$

The LRT and the corresponding p-value are denoted in Table 3. In this case, the calculated LRT statistic is greater than the critical point for this test, which is very small. According to the LRT, we conclude that this data fts the XGLo much better than the Lomax distribution.

Table 3: KS test, LRT for Lomax and XGLo models with different data sets

data		XGLo	Lo	LRT	P-Value
I	LogL	410.7478	413.8988	6.3021	0.0121
	df	3	2	1	
	KSD	0.0724	0.1033		
	PVKS	0.5139	0.1305		
II	LogL	700.3959	703.7217	6.6517	0.0099
	df	3	2	1	
	KSD	0.0626	0.0925		
	PVKS	0.7870	0.3081		
III	LogL	153.8994	161.9805	16.1622	0.0001
	df	3	2	1	
	KSD	0.1095	0.2177		
	PVKS	0.5503	0.0147		

Figure 3, 4 and 5 shows the fit of the empirical CDF, histogram and PP-plot as follows

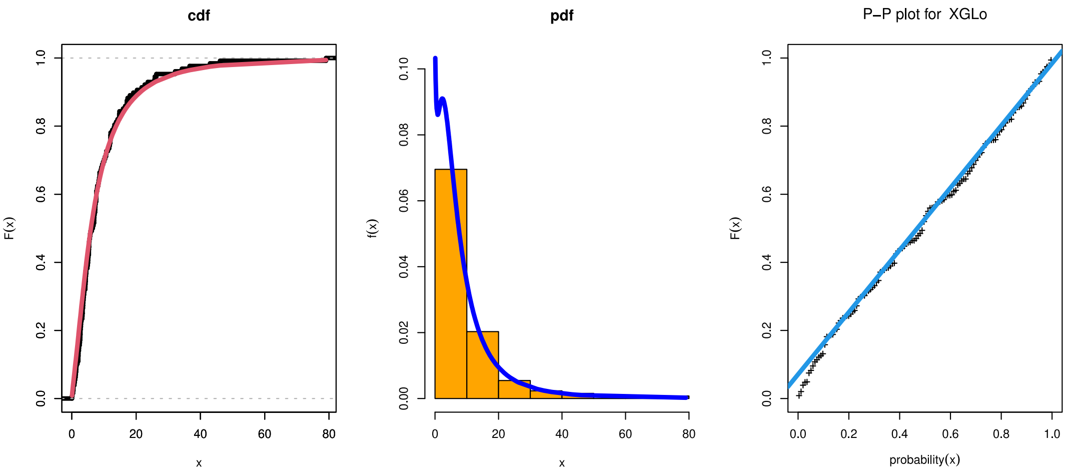

Figure 3. Cumulative function with empirical CDF, histogram with the Fitted pdf of XGLo distribution, and P-P plot for the XGLo distribution for data set I

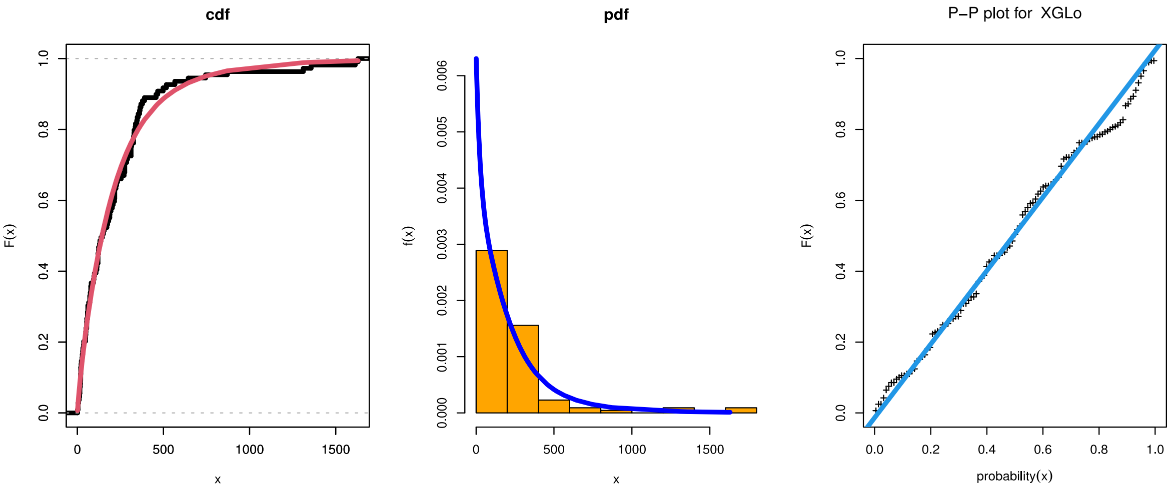

Figure 4. Cumulative function with empirical CDF, histogram with the Fitted pdf of XGLo distribution, and P-P plot for the XGLo distribution for data set II

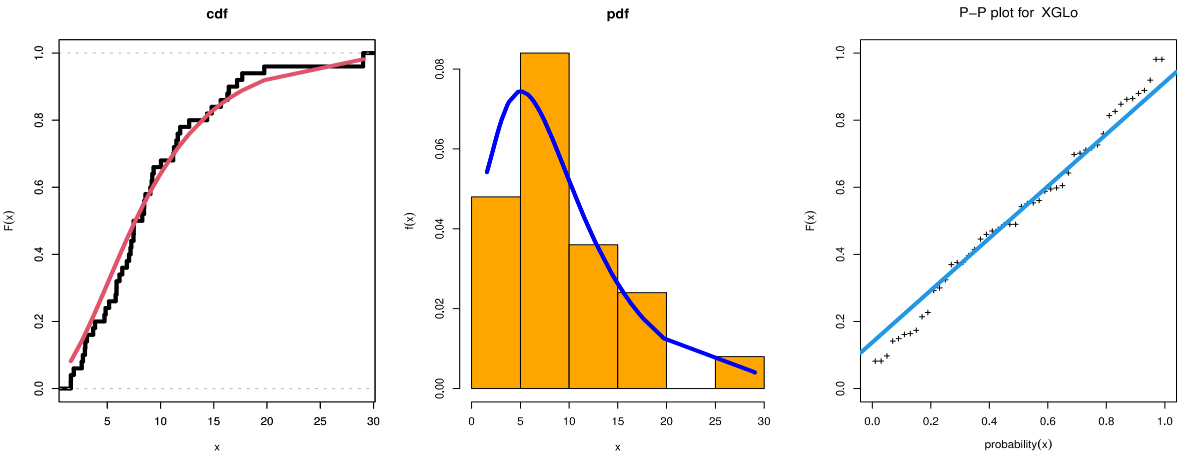

Figure 5. Cumulative function with empirical CDF, histogram with the Fitted pdf of XGLo distribution, and P-P plot for the XGLo distribution for data set III

7. Conclusion

The X-Gamma Lomax (XGLo) distribution, a new extension of the Lomax distribution, is a new three-parameter model that we propose in this study. The widespread application of the Lomax model in life testing serves as the driving force behind the XGLo distribution, which offers greater flexibility when analysing lifetime data. MLE and MPS are used to derive the XGLo distribution parameter estimation. The model parameters are estimated using estimation techniques, and the model performance is evaluated using simulation results. The proposed model, which is based on three real-world data, regularly offers a better fit than the Lomax distributions.

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