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Research article

Statistical Properties of Weighted Shanker Distribution

Tamer S. Helal [1], Ahmed M. Elsehetry [2*], Rasha S. Elshaarawy [3]

[1] Department of Mathematical Statistics Faculty of Graduate Studies for Statistical Research, Cairo University, Giza, Egypt, tamerhelal2006@yahoo.com.
[2] Department of Mathematical Statistics Faculty of Graduate Studies for Statistical Research, Cairo University, Giza, Egypt, ah_sehetry@hotmail.com.
[3] Department of Mathematical Statistics Faculty of Graduate Studies for Statistical Research, Cairo University, Giza, Egypt, Rashasaber123@hotmail.com.
*** Correspondence:** ah_sehetry@hotmail.com.

Abstract: In this article we introduce and investigate the weighted Shanker distribution, a weighted version of Shanker's (2015) Shanker distribution. To obtain the form of the weighted Shanker distribution, which is shown as a generalization of the Shanker distribution, a special non-negative weight function is considered. The statistical properties of the weighted Shanker distribution are investigated. We propose a maximum likelihood method for estimating the weighted version's unknown parameter. To observe the pattern of estimates for different sample sizes, a sample generation algorithm and a Monte Carlo simulation study are prepared.

Keywords: Weighted Distributions; Maximum Likelihood Estimation; Shanker Distribution; Survival function; Hazard rate function.

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1. Introduction

The theory of weighted distributions offers a group approach to model specification and appropriate statistical problems. It offers a method for fitting models to unknown weight functions when samples are drawn from both the original and established distributions. Weighted distributions account for the procedure of ascertainment by adjusting the probabilities of events' actual occurrence to arrive at a specification of those events' probabilities as observed and recorded. Weighted distributions are frequently used in studies relating to reliability, analysis of family data, Meta-analysis and analysis of

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intervention data, biomedicine, ecology, and some other areas for the improved performance of appropriate statistical models.

Fisher and Rao (1934) introduced the idea of weighted distributions (1965). Fisher (1934) investigated how ascertainment methods can influence the shape of the distribution of recorded observations, and Rao (1965) introduced and formulated it in general terms in connection with modeling statistical data when the standard practice of using standard distributions was found to be unsuitable. Weighted distributions are used to modulate the probabilities of observed and transcribed events. Patil and Rao (1978) introduced some useful ideas. We would like to draw attention to the works of Gupta and Keating (1985), Gupta and Kirmani (1990), Oluyede (1999), and their references.

Suppose X is a non-negative continuous random variable with probability density function (pdf)f(x). The pdf of the weighted random variable X_W is given by:

$$f_w(x) = \frac{w(x)\, f(x)}{W_D}, \qquad x>0, \tag{1.1}$$

where, $W_D = \int_0^\infty w(x)\, f(x)\, dx,\quad x>0,$ and w(x) is a non-negative weight function.

Note that similar definition can be stated for the discrete random variables. When we use weighted distributions as a tool in the selection of suitable models for observed data it is the choice of the weight function that fits the data.

Weighted distributions appear frequently in research concerning reliability, biomedicine, and ecology, as evidenced by Patil and Rao (1978), Gupta and Kirmani (1990), Gupta and Keating (1985), Oluyede (1999), and references therein. Zelen (1974) invented weighted distributions to comprise what he widely perceived as length-biased sampling (introduced earlier in Cox (1962)) in the context of cell kinetics and early disease detection. See Rao (1997), Patil and Ord (1997), Zelen and Feinleib (1969), El-Shaarawi and Walter (1969) for additional significant outcomes on weighted distributions (2002). Priyadarshani (2011) developed a new class of weighted generalized gamma distribution and relevant distribution, theoretical features of the generalized gamma model, Jing (2010) presented the weighted inverse Weibull distribution and beta-inverse Weibull distribution, theoretical properties of them, and there are many research studies for weighted distribution. Castillo and Perez-Casany (1998) developed new exponential families derived from the weighted distribution concept, which include and generalize the Poisson distribution. Shaban and Boudrissa (2007) demonstrated that the length-biased version of the Weibull distribution known as the Weibull Length-biased (WLB) distribution is unimodal throughout examining its shape, with the other characteristics, Das and Roy (2011 a) addressed the length-biased Weighted Generalized Rayleigh distribution with its characteristics, and they are developing the length-biased version of the weighted Weibull distribution see Das and Roy (2011 b). On a Length-Biased Weighted Weibull DistributionPatil and Ord (1976) defined size-biased sampling and weighted distributions by identifying situations in which the fundamental models preserve their form. See also (Oluyede and George (2002), Ghitany and Al-Mutairi (2009), Ahmed, Reshi, and Mir (2013), Almetwally et al. (2022), Broderick X. S., Oluyede and Pararai (2012), Oluyede and Terbeche M (2007) for much more essential weighted distribution outcomes).

In this paper is to introduce the new class of weighted distributions which is given by:

$$f_w(x)=\frac{g(x)\,f(x)}{W_D}, \qquad x>0, \tag{1.2}$$

where w(x) = g(x) is pdf and $W_D=\int_0^\infty g(x)\,f(x)\,dx, \qquad x>0.$

Shanker (2015) introduced a lifetime distribution named Shanker distribution having probability density function (pdf).

$$f(x;\theta)=\frac{\theta^2}{\theta^2+1}(\theta+x)e^{-\theta x}; \qquad x>0,\theta>0. \tag{1.3}$$

The pdf shows that the Shanker distribution is a two – component mixture of an exponential distribution (with scale parameter) and a gamma distribution (with shape parameter 2 and scale parameter). That is, Shanker distribution is a convex combination of exponential (θ) and gamma (2, θ) distributions.

The corresponding cumulative distribution function (cdf) of Shanker distribution is given by:

$$F(x;\theta)=1-\left[1+\frac{\theta x}{\theta^2+1}\right]e^{-\theta x}; \qquad x>0,\theta>0.$$

Our aim of this work is to introduce a new class of weighted distributions depending on w(x) = g(x) is any non negative pdf. The rest of the paper organized as follows: In the next section, the new weighted Shanker distribution in Section 2. Study some general mathematical properties of the new distribution in Section 3. In Section 4, estimation of the parameters of the NWS model is implemented through maximum likelihood method. Simulation study is carried out to evaluate the performance of the unknown parameters in Section 5. Finally, concluding remarks are handled in Section 6.

2. A New Weighted Shanker Distribution:

In this section, the new distribution, called the new weighted Shanker distribution we can get it by Consider the weight function is the exponential distribution

$$g(x;\lambda)=\lambda e^{-\lambda x}; \qquad x>0;\lambda>0, \tag{2.1}$$

and

$$W_D = \frac{\theta^2 \lambda}{\theta^2 + 1}\int_0^\infty (\theta + x) e^{-(\lambda+\theta)x} dx \tag{2.2}$$

Hence, the pdf of the new weighted Shanker distribution (NWSD) is derived by substituting (1.3), (2.1) and (2.2) into (1.2) to give:

$$f_{NWSD}(x;\lambda,\theta) = \frac{(\lambda+\theta)^2}{\theta^2 + \lambda\theta + 1}(\theta + x)e^{-(\lambda+\theta)x}, \tag{2.3}$$

and the cdf of NWSD is:

$$F_{NWSD}(x;\lambda,\theta) = 1 - \left(1 + \frac{(\lambda+\theta)x}{\theta^2 + \lambda\theta + 1}\right)e^{-(\lambda+\theta)x}, \tag{2.4}$$

The Surviva function, hazard rate function, reversed hazard rate function, cumulative hazard rate function is given respectively

$$\overline{F}_w(x) = 1 - F_w(x) = \left(1 + \frac{(\lambda+\theta)x}{\theta^2 + \lambda\theta + 1}\right)e^{-(\lambda+\theta)x} ,$$

$$h(x) = \frac{f_w(x)}{\overline{F}_w(x)} = \frac{(\lambda+\theta)^2(\theta + x)}{(\theta^2 + \lambda\theta + 1)\left(1 + \frac{(\lambda+\theta)x}{\theta^2 + \lambda\theta + 1}\right)} ,$$

$$\tau(x) = \frac{f_w(x)}{F_w(x)} = \frac{(\lambda+\theta)^2(\theta + x)e^{-(\lambda+\theta)x}}{(\theta^2 + \lambda\theta + 1)\left[1 - \left(1 + \frac{(\lambda+\theta)x}{\theta^2 + \lambda\theta + 1}\right)e^{-(\lambda+\theta)x}\right]} ,$$

and

$$H(x) = -\ln\left(\overline{F}_w(x)\right) = -\ln\left(\left(1 + \frac{(\lambda+\theta)x}{\theta^2 + \lambda\theta + 1}\right)e^{-(\lambda+\theta)x}\right).$$

Plots of the NWSD pdf and hazard rate function for some parameter values are displayed in Figures 1 and 2 respectively.

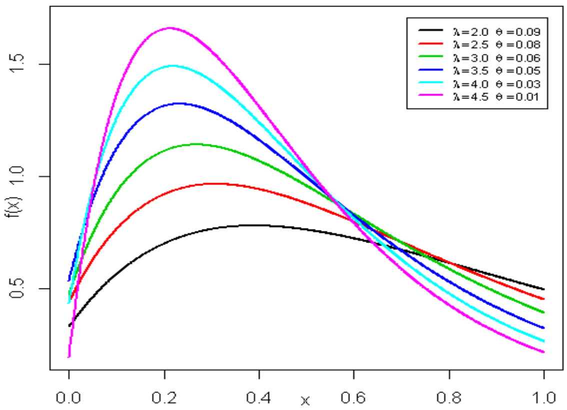

Figure1. The pdf of *NWSD* for some different parameter values

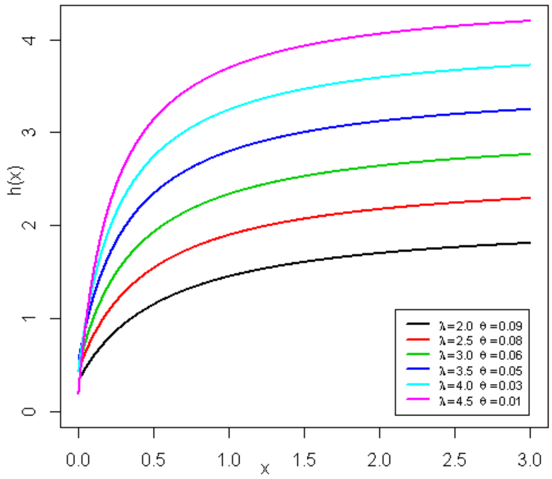

Figure 2. Hazard rate functions of $NWSD$ for some different parameter values

Note that: the new weighted Shanker is very flexible model. The NWS distribution contains special-models if $\lambda = 0$ we get Shanker distribution.

3. Statistical Properties:

The statistical characteristics of the NWS distribution, particularly the quantile function, moments, and moment generating function, are studied in this section.

3.1 Quntile Function

Let $\sim \mathrm{NWS}(\lambda, \theta)$, the quantile function, say $\mathrm{Q(u)}$, is defined by $\mathrm{F}\big(\mathrm{Q(u)}\big) = \mathrm{u}$. Then, we can obtain $\mathrm{Q(u)}$ as the root of the following equation

$$-(\theta^2+\lambda\theta+1+(\lambda+\theta)Q(u))e^{-(\lambda+\theta)Q(u)}=(u-1)(\theta^2+\lambda\theta+1)$$

for $0 < \mathrm{p} < 1$. Substituting $Z(u)=-(\theta^2+\lambda\theta+1+(\lambda+\theta)Q(u))e^{-(\lambda+\theta)Q(u)}$ we can rewrite it as

$$Z(u)e^{Z(u)}=(u-1)\left(\theta^2+\lambda\theta+1\right)e^{-\left(\theta^2+\lambda\theta+1\right)}$$

Hence, the equation of $\mathrm{Z(u)}$ is

$$Z(u)=W\left((u-1)\left(\theta^2+\lambda\theta+1\right)e^{-\left(\theta^2+\lambda\theta+1\right)}\right)$$

where W[.] is the Lambert function (Corless et al., 1996). Then, we obtain

$$Q(u)=-\frac{\theta^2+\lambda\theta+1}{\lambda+\theta}-\frac{W\left((u-1)\left(\theta^2+\lambda\theta+1\right)e^{-\left(\theta^2+\lambda\theta+1\right)}\right)}{\lambda+\theta}. \tag{3.1}$$

Specifically, the first quartile, the median, and the third quartile are obtained by setting u=0.25, 0.5 and 0.75, respectively, in the previous equation.

3.2 Moments and Moment Generating function:

Theorem (3.1): If X has $NWS(x,\lambda,\theta)$, then the r^{th} moment of X is given by the following:

$$\mu_r' = \frac{\left(\theta^2+\lambda\theta+r+1\right)\Gamma\left(r+1\right)}{\left(\lambda+\theta\right)^r\left(\theta^2+\lambda\theta+1\right)}, \tag{3.2}$$

Proof: The proof can be easily achieved by suing the definition r^{th} ordinary moment,

$$\mu_r' = E\left(X^r\right) = \int_0^\infty x^r f(x)dx = \frac{\left(\lambda+\theta\right)^2}{\theta^2+\lambda\theta+1}\left[\theta\int_0^\infty x^r e^{-(\lambda+\theta)x}dx + \int_0^\infty x^{r+1}e^{-(\lambda+\theta)x}dx\right].$$

The first four moments is given respectively

$$\mu_1^{\backslash} = \frac{\left(\theta^2+\lambda\theta+2\right)}{\left(\lambda+\theta\right)\left(\theta^2+\lambda\theta+1\right)}$$

$$\mu_2^{\backslash} = \frac{2\left(\theta^2+\lambda\theta+3\right)}{\left(\lambda+\theta\right)^2\left(\theta^2+\lambda\theta+1\right)}$$

$$\mu_3^{\backslash} = \frac{6\left(\theta^2+\lambda\theta+4\right)}{\left(\lambda+\theta\right)^3\left(\theta^2+\lambda\theta+1\right)}$$

and

$$\mu_4^{\backslash} = \frac{24\left(\theta^2+\lambda\theta+5\right)}{\left(\lambda+\theta\right)^4\left(\theta^2+\lambda\theta+1\right)}.$$

According to the first four moments of the NWS distribution, the measures of skewness $A(\Phi)$ and kurtosis $k(\Phi)$ of the NWS distribution can be computed as below:

$$A(\Phi) = \frac{\mu_3(\theta)-3\mu_1(\theta)\mu_2(\theta)+2\mu_1^3(\theta)}{\left[\mu_2(\theta)-\mu_1^2(\theta)\right]^{\frac{3}{2}}}, \tag{3.3}$$

and

$$k(\Phi) = \frac{\mu_4(\theta)-4\mu_1(\theta)\mu_3(\theta)+6\mu_1^2(\theta)\mu_2(\theta)-3\mu_1^4(\theta)}{\left[\mu_2(\theta)-\mu_1^2(\theta)\right]^2}. \tag{3.4}$$

Theorem (3.2): If X has NWS distribution, then the moment generating function $M_X(t)$ has the next form:

$$M_X(t)=\frac{\theta^2+\lambda\theta-\theta t+1}{\left(\theta^2+\lambda\theta+1\right)\left(\lambda+\theta-t\right)^2}, \tag{3.5}$$

Proof: The proof is simple to accomplish by employing the concept of the moment generating function, which is provided via:

$$M_X(t)=E\left(e^{tX}\right)=\int_0^\infty e^{tx} f_{NCWS}(x)dx=\frac{(\lambda+\theta)^2}{\theta^2+\lambda\theta+1}\int_0^\infty(\theta+x)e^{-(\lambda+\theta-t)x}dx$$

3.3 Incomplete Moments and Conditional Moments:

The incomplete moments, say$\varphi_s(t)$, is given by:

$$\begin{aligned}\phi_s(t)&=\int_0^t x^s f_{NWS}(x)dx\\&=\frac{(\lambda+\theta)^2}{\theta^2+\lambda\theta+1}\left[\frac{\theta\gamma\left(s+1,(\lambda+\theta)t\right)}{(\lambda+\theta)^{s+1}}+\frac{\gamma\left(s+2,(\lambda+\theta)t\right)}{(\lambda+\theta)^{s+2}}\right],\end{aligned} \tag{3.6}$$

Theorem (3.3): If X has $NCWS$ distribution, then the conditional moments $\tau_s(t)$ has the following form

$$\tau_s(t)=\frac{(\lambda+\theta)^2}{\theta^2+\lambda\theta+1}\left[\frac{\theta\Gamma\left(s+1,(\lambda+\theta)t\right)}{(\lambda+\theta)^{s+1}}+\frac{\Gamma\left(s+2,(\lambda+\theta)t\right)}{(\lambda+\theta)^{s+2}}\right], \tag{3.7}$$

Proof: The conditional moments, say $\tau_s(t)$, is given by**:**

$$\tau_s(t)=\int_t^\infty x^s f_{NWS}(x)dx.$$

The following formula can be used to prove the result:

$$\tau_s(t)=\frac{(\lambda+\theta)^2}{\theta^2+\lambda\theta+1}\int_t^\infty\left(\theta x^s+x^{s+1}\right)e^{-(\lambda+\theta)x}dx.$$

3.4 Inequality Measures:

The Lorenz and Bonferroni curves are among the most commonly utilized inequality indicators in income and wealth distribution Kleiber (1999). Zenga introduced the Zenga curve (2007). In this section, we will calculate the Lorenz, Bonferroni, and Zenga curves for the NCWS distribution. The Lorenz, Bonferroni, and Zenga curves are described as follows

$$3.5\ \ L_F(x)=\frac{\int_0^t xf(x)dx}{E(X)}=\frac{(\lambda+\theta)^3}{\theta^2+\lambda\theta+2}\left[\frac{\theta\gamma(2,(\lambda+\theta)t)}{(\lambda+\theta)^2}+\frac{\gamma(3,(\lambda+\theta)t)}{(\lambda+\theta)^3}\right],$$

$$B_F(x)=\frac{\int_0^t xf(x)dx}{E(x)F(x)}=\frac{L_F(x)}{F(x)}=\frac{(\lambda+\theta)^3\left[\frac{\theta\gamma(2,(\lambda+\theta)t)}{(\lambda+\theta)^2}+\frac{\gamma(3,(\lambda+\theta)t)}{(\lambda+\theta)^3}\right]}{(\theta^2+\lambda\theta+2)\left(1-\left(1+\frac{(\lambda+\theta)x}{\theta^2+\lambda\theta+1}\right)\right)e^{-(\lambda+\theta)x}},$$

and $A_F(x)=1-\frac{\mu^-(x)}{\mu^+(x)}$,

where, $\mu^-(x)=\frac{\int_0^t xf(x)dx}{E(x)}=\frac{(\lambda+\theta)^3}{\theta^2+\lambda\theta+2}\left[\frac{\theta\gamma(2,(\lambda+\theta)t)}{(\lambda+\theta)^2}+\frac{\gamma(3,(\lambda+\theta)t)}{(\lambda+\theta)^3}\right]$

and

$$\mu^+(x)=\frac{\int_0^t xf(x)dx}{1-F(x)}=\frac{(\lambda+\theta)^3\left[\frac{\theta\Gamma(2,(\lambda+\theta)t)}{(\lambda+\theta)^2}+\frac{\theta\Gamma(3,(\lambda+\theta)t)}{(\lambda+\theta)^3}\right]}{(\theta^2+\lambda\theta+2)\left(1+\frac{(\lambda+\theta)x}{\theta^2+\lambda\theta+1}\right)e^{-(\lambda+\theta)x}}$$

respectively.

4. Maximum Likelihood Estimation

In this section, we determine the maximum likelihood estimates (MLEs) of the parameters of the NWS distribution from complete samples only. Let $X_1, X_2, \dots, X_n$ be a random sample of size n from $NWS(x,\lambda,\theta)$. The log-likelihood function for the vector of parameters $\Phi=(x,\lambda,\theta)$ is given by:

$$\ln L = 2n\ \ln(\lambda+\theta) - n\ \ln\left(\theta^2+\lambda\theta+1\right) + \sum_{i=1}^{n}\ln(\theta+x_i) - (\lambda+\theta)\sum_{i=1}^{n}x_i, \tag{4.1}$$

The log-likelihood can be maximized either directly or by solving the nonlinear likelihood equations obtained by differentiating (4.1). The components of the score vector are given by:

$$\frac{\partial \ln L}{\partial \theta} = \frac{2n}{\lambda+\theta} - \frac{n(2\theta+\lambda)}{\theta^2+\lambda\theta+1} + \sum_{i=1}^{n}\frac{1}{\theta+x_i} - \sum_{i=1}^{n}x_i, \tag{4.2}$$

and

$$\frac{\partial \ln L}{\partial \lambda} = \frac{2n}{\lambda+\theta} - \frac{n\theta}{\theta^2+\lambda\theta+1} - \sum_{i=1}^{n}x_i, \tag{4.3}$$

We can use the maximum likelihood method to estimate the unknown parameters by setting the above non-linear equations to zero and solving them simultaneously. As a result, we must use a mathematical package to obtain the MLE of the unknown parameters.

5. Simulation

Comparing the theoretical performances of different estimators (MLE) for the NWS distribution is extremely difficult. As a result, simulation is required to compare the estimation performances, particularly their biases and mean square errors for different sample sizes. Mathematica 9 software is used to conduct a numerical analysis. The experiments consider different sample sizes of n = 20, 30, 50, and 100. Furthermore, the various values of the parameters λ and θ. The experiment will be repeated a thousand times. The maximum likelihood method will be used to estimate the parameters in each experiment. These experiments will provide means, MSEs, and biases for the various estimators.

Table 1. MLEs, biases and MSEs for some parameter values

n	Par	Init	MLE	Bais	MSE	Init	MLE	Bais	MSE
20	λ	3.00	3.6914	0.6914	1.0980	4.00	3.2192	-0.7808	7.0204
	θ	0.07	0.0061	-0.0639	0.0116	0.07	0.7907	0.7207	2.9668
30	λ	3.00	3.1185	0.1185	0.8434	4.00	4.1532	0.1532	1.7976
	θ	0.07	0.1551	0.0851	0.0852	0.07	0.2271	0.1571	0.1381
50	λ	3.00	3.0984	0.0984	0.3072	4.00	4.1244	0.1244	1.0665
	θ	0.07	0.0580	-0.0120	0.0037	0.07	0.1721	0.1021	0.0670
100	λ	3.00	3.0266	0.0266	0.0967	4.00	4.1722	0.1722	0.6271
	θ	0.07	0.0403	-0.0297	0.0022	0.07	0.0950	0.0250	0.0086

Continued of Table 1

n	Par	Init	MLE	Bais	MSE	Init	MLE	Bais	MSE
20	λ	5.0	5.0204	0.0204	6.8155	3.00	2.5742	-0.4258	3.6407
	θ	0.07	0.3663	0.2963	0.4584	0.05	0.4667	0.4167	1.3705
30	λ	5.0	5.5500	0.5500	0.7369	3.00	3.1139	0.1139	0.5694
	θ	0.07	0.0480	-0.0220	0.0078	0.05	0.0611	0.0111	0.0187
50	λ	5.0	4.5628	-0.4372	0.8624	3.00	2.6919	-0.3081	0.2556
	θ	0.07	0.1335	0.0635	0.0211	0.05	0.1043	0.0543	0.0105
100	λ	5.0	5.0220	0.0220	0.1949	3.00	3.0273	0.0273	0.0780
	θ	0.07	0.0715	0.0015	0.0019	0.05	0.0481	-0.0019	0.0006
20	λ	4.0	4.3620	0.3620	2.3352	5.0	5.7278	0.7278	3.4731
	θ	0.05	0.0813	0.0313	0.0684	0.05	0.0367	-0.0133	0.0043
30	λ	4.0	4.1362	0.1362	0.7124	5.0	5.4243	0.4243	0.9436
	θ	0.05	0.0902	0.0402	0.0104	0.05	0.0623	0.0123	0.0055
50	λ	4.0	3.8237	-0.1763	0.248813	5.0	5.2233	0.2233	0.235436
	θ	0.05	0.0597	0.0097	0.003937	0.05	0.0248	-0.0252	0.004065
100	λ	4.0	4.0952	0.0952	0.1783	5.0	5.1406	0.1406	0.1873
	θ	0.05	0.0633	0.0133	0.0018	0.05	0.0519	0.0019	0.0013

6. Conclusion

In this article we introduce the weighted Shanker distribution. Different statistical properties of weighted Shanker distribution are studied. For the purpose of estimating the unknown parameter of the weighted version, we suggest the maximum likelihood method. The pattern of the estimates for various sample sizes is created using a sample generation technique and Monte Carlo simulation research.

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الخصائص الاحصائية لتوزيع شانكر المرجح

تامر سيد هلال [1]، أحمد محمد السحتري [1]، رشا صابر الشعراوي [1]

[1] قسم الإحصاء الرياضي، كلية الدراسات العليا للبحوث الإحصائية، جامعة القاهرة، الجيزة ، مصر.

الملخص: في هذه المقالة نقدم توزيع **Shanker** المرجح ، وهو نسخة مرجحة من توزيع .Shanker (2015) للحصول على شكل توزيع شانكر المرجح ، والذي يظهر كتعميم لتوزيع شانكر ، يتم أخذ دالة مرجحة خاصة غير سلبية. تم دراسة الخصائص الإحصائية لتوزيع شانكر المرجح. سيتم استخدام طريقة الامكان الأعظم لتقدير المعالم غير المعروفة للتوزيع المرجح. لملاحظة نمط التقديرات لأحجام العينات المختلفة ، تم إعداد خوارزمية توليد العينة ودراسة المحاكاة باستخدام طريقة مونت كارلو.

الكلمات الافتتاحية: التوزيعات المرجحة، تقدير الامكان الأعظم، توزيع شانكر، دالة البقاء، دالة الفشل