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Research article

Bayesian and E-Bayesian Estimation for Odd Generalized Exponential Inverted Weibull Distribution

A. A. Mohamed[1, *], R. M. Refaey[2] and, G. R. AL-Dayian[3]
[1]Statistics Department, Faculty of Commerce, AL-Azhar University (Girls' Branch), Tafahna Al-Ashraf, Egypt; statistic_prof121@hotmail.com;
[2]Statistics Department, Faculty of Commerce, AL-Azhar University (Girls' Branch), Cairo, Egypt; reda_2222@outlook.com;
[3]Statistics Department, Faculty of Commerce, AL-Azhar University (Girls' Branch), Cairo, Egypt; aldayian@yahoo.com

* **Correspondence**: statistic_prof121@hotmail.com

Abstract: Lifetime distributions under the Type-II censored scheme have been attracting great interest due to their wide application in the fields of science, reliability, economics, environmental sciences, finance, engineering, social sciences, medicine, and other fields. In this paper, Bayesian and E-Bayesian estimators for the shape parameters of the odd generalized exponential inverted Weibull distribution are estimated. The Bayes and E-Bayes estimators are derived under the balanced squared error loss function as a symmetric loss function, and the balanced linear exponential loss function as an asymmetric loss function, based on a Type II censored sample. Based on informative gamma priors and uniform hyper-prior distributions, the estimators are obtained. Finally, the performance of the proposed Bayes and E-Bayes estimates is evaluated through a simulation study to show the high flexibility and potential applications of the distribution. Moreover, the results are applied to three real data sets from the COVID-19 death rate in different countries.

Keywords: Odd generalized exponential inverted Weibull distribution; Type-II censored samples; Bayesian estimation; E-Bayesian estimation; balanced loss function.

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1.Introduction

Many generalized distributions have been put out recently, and when applied to actual data, their

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level of flexibility over their baseline distributions has been demonstrated. In various applied fields, including reliability, engineering, economics, environmental sciences, and medicine, statisticians are becoming more interested in adding one or more parameters to a baseline distribution since it offers them significant modelling freedom.

There are numerous instances in reliability studies and life testing where units are lost or taken out of the test before it fails. In an industrial experiment, for instance, equipment can break by accident; in a clinical trial, participants might withdraw from the study; or they might have to end early for budgetary reasons. Because of the time and expense constraints of the experiment, it is common practice in many cases to remove units prior to failure. These tests' or experiments' data are referred to as "censored data."

Several methods of censoring have been suggested in the literature to analyze lifetime data. Type I and Type-II censoring schemes have probably found the most extensive applications in these situations. It saves time and money, since if a total of n items is placed on a test, then continuing until all n items failed, could take a very long time and cost much money. Hence, the test is terminated before the failure of all items. The total number of the sample items is known, but the measurements on some of which are not observed such sample is called censored sample. [For more details, see, Mann *et al*. (1974), Nelson (1982), and Lawless (2003)]. In Type-I censoring a life test is conducted for a fixed-time period while in Type-II censoring an experiment terminates when a prescribed number of units fail.

The Bayesian approach the unknown parameters as random variables and relies on knowledge about the parameters already known .Also, provides some precise advantages when the sample size is limited. Objective Bayes estimates can be derived using non-informative priors, like the Jeffreys prior, when little information about the prior information is provided. The Bayes estimators of the parameters are built using both symmetric and asymmetric loss function functions. For a few essential sources, see the works of Jeffreys (1998) and Xu and Tang (2011).

Han (2007) proposed the *expected Bayesian* (E-Bayesian) estimation method which is very simple and it’s a special Bayesian method used in the area related for the life testing of products with high reliability, small sample size, or censored data. It is now more widely accepted. Numerous authors applied the E-Bayesian approach to a variety of distributions, for instance, Reyad and Ahmed (2015), Gupta (2017), Han (2019), Algarni *et al.* (2020), Han (2020), Okasha (2020) and Rabie and Li (2020).

The Bayesian and E-Bayesian estimation were examined by numerous scholars. For instance, Yahgmaei *et al.* (2013) obtained the Bayesian estimation of the scale parameter of inverse Weibull distribution under the asymmetric loss functions. Reyad *et al.* (2016) applied the Bayesian and E-Bayesian estimation for the Kumaraswamy distribution based on Type-II censoring. Wei *et al.* (2017) proposed the Bayes estimation of the Lomax distribution parameter in the composite LINEX loss of symmetry. Algarni *et al.* (2020) investigated the E-bayesian estimation of chen distribution based on Type-I censoring scheme. Okasha (2021) discussed the E-Bayesian estimation of reliability characteristics of a Weibull distribution with applications. Athirakrishnan and Abdul-Sathar (2022) introduced the E-Bayesian and hierarchical Bayesian estimation of inverse Rayleigh distribution. Al Mutairi *et al.* (2023) presented the Bayesian and E-Bayesian estimation based on constant-stress partially accelerated life testing for inverted Topp–Leone distribution. Additionally, Nagy (2023) investigated the Bayesian and E-Bayesian estimation for Rayleigh distribution using unified progressive hybrid censored samples. Muhammed and Almetwally (2024) constructed the Bayesian and non-Bayesian estimation for the shape parameters of new versions of bivariate inverse Weibull distribution based on progressive Type II censoring. Additionally, Hasaballah *et al.* (2024) studied the non-Bayesian and Bayesian estimation for Lomax distribution under randomly censored with application.

Gangopadhyay *et al.* (2024) studied the Bayesian inference on parameters and reliability characteristics for inverse Xgamma distribution under adaptive-general progressive Type-II censoring. El-Morshedy (2024) derived the classical and Bayesian techniques for modelling engineering dataset using new generalized probability distribution with mathematical features. Almuqrin (2024) proposed the Bayesian and non-Bayesian inference for the compound Poisson log-normal model with application in finance. AL-Essa *et al.* (2024) introduced Bayesian estimation of the Pareto model based on Type-II censoring data by employing non-linear programming. Athirakrishnan and Abdul-Sathar (2024) considered the E-Bayesian and hierarchical Bayesian estimation for inverse Rayleigh distribution based on left censoring scheme. Hassan *et al.* (2024) constructed the classical and Bayesian inference for the length biased weighted Lomax distribution under progressive censoring scheme.

The rest of this paper is organized as follows: in Section 2, presents a brief summary of the OGE-IW(α, β, ζ) distribution and descriptions of the main its properties. The Bayes estimators for the unknown parameters of the OGE-IW distribution based on Type-II censored samples under the *balanced squared loss* (BSEL) and *balanced linear exponential loss* (BLL) functions are obtained in Section 3. The E-Bayes estimators for the unknown parameters of the OGE-IW distribution under the BSEL and BLL functions are discussed in Section 4. A numerical example is given to illustrate the theoretical results and an application using real data sets are used to demonstrate how the results can be used in practice in Section 5. Finally, some general conclusions are introduced in Section 6.

2. Odd Generalized Exponential Inverted Weibull Distribution

The inverted Weibull distributions have great importance due to their applicability in many areas such as engineering discipline of reliability, biological sciences, life test problems, medical, etc. The inverted Weibull distribution can be used to a diverse model of failure characteristics, such as infant mortality, age of production, and periods of erosion. The inverted Weibull distribution can also be used to determine the cost-effectiveness and maintenance periods of reliability centered maintenance activities.

Hassan *et al*. (2018) constructed a distribution with three parameters and presented some mathematical statistical properties of OGE-IW distribution such as quantile function, mode, moments, probability-weighted moments, incomplete moments, stress-strength model, moments of residual life function, and Rényi entropy. Also, they provided graphical illustrations of the dimensions of OGE-IW distribution and estimated the parameters using the ML method based on complete samples.

Assume a random variable X that follows the OGE-IW distribution, denoted by X ~ OGE-IW(α,β,ζ). The pdf, cdf, sf, hrf and rhrf of OGE-IW distribution are respectively written as:

$$f(x;\underline{\Psi}) = \alpha\beta\zeta x^{-\alpha-1}e^{-\beta x^{-\alpha}}\left(1-e^{-\beta x^{-\alpha}}\right)^{-2}\exp\left[-\left(\frac{\zeta}{e^{\beta x^{-\alpha}}-1}\right)\right],\quad x>0;\ (\underline{\Psi}>\underline{0}), \tag{1}$$

and

$$F(x;\underline{\Psi}) = 1-\exp\left[-\left(\frac{\zeta}{e^{\beta x^{-\alpha}}-1}\right)\right],\qquad x>0;(\underline{\Psi}>\underline{0}), \tag{2}$$

where $\underline{\Psi}=(\alpha,\beta,\zeta)$, α,β are the shape parameters and ζ is the scale parameter.

The pdf of OGE-IW distribution can be symmetric, unimodal and right skewed. For α =2, the OGE-IW $(\underline{\Psi})$distribution reduces to a new model named as odds generalized exponential inverse Rayleigh distribution. For $\alpha=1$, the OGE-IW $(\underline{\Psi})$distribution reduces to another new model named as odds generalized exponential inverse exponential distribuiton.

The sf and hrf of OGE-IW $(\underline{\Psi})$distribution are, respectively, given by

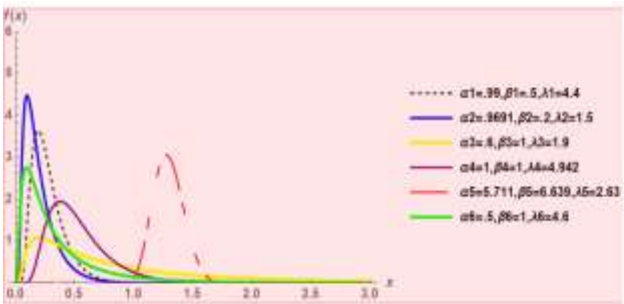
$$s(x;\underline{\Psi}) = \exp\left[-\left(\frac{\zeta}{e^{\beta x^{-\alpha}}-1}\right)\right]\qquad x>0;\ (\underline{\Psi}>\underline{0}), \tag{3}$$

$$h(x;\underline{\Psi}) = \zeta\alpha\beta x^{-\alpha-1}e^{-\beta x^{-\alpha}}\left(1-e^{-\beta x^{-\alpha}}\right)^{-2},\qquad x>0;\ (\underline{\Psi}>\underline{0}), \tag{4}$$

and

$$rh(x;\underline{\Psi}) = \frac{\alpha\beta\zeta x^{-\alpha-1}e^{-\beta x^{-\alpha}}\left(1-e^{-\beta x^{-\alpha}}\right)^{-2}}{\left[\exp\left(\frac{\zeta}{e^{\beta x^{-\alpha}}-1}\right)-1\right]},\qquad x>0;\ (\underline{\Psi}>\underline{0}), \tag{5}$$

The plots of the pdf, hrf and rhrf of OGE-IW $(\underline{\Psi})$ distribution are provided for different values of parameters in Figure 1 and Figure 6.



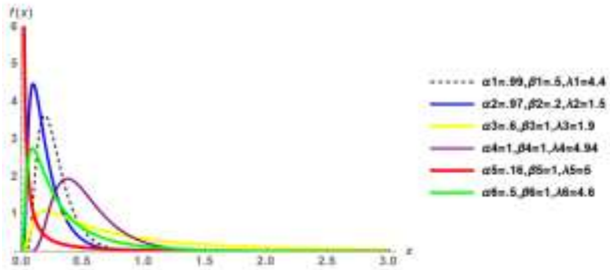
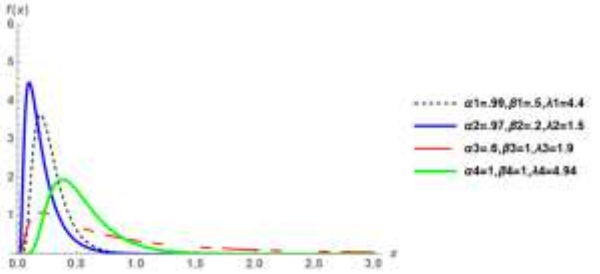


Figure1 Figure2



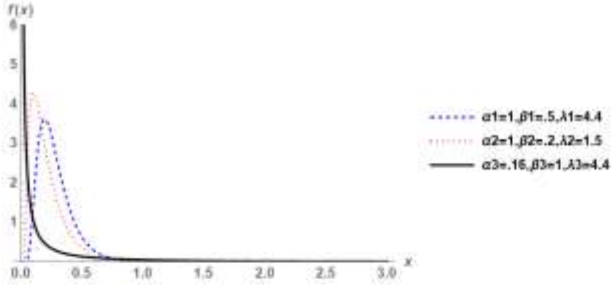
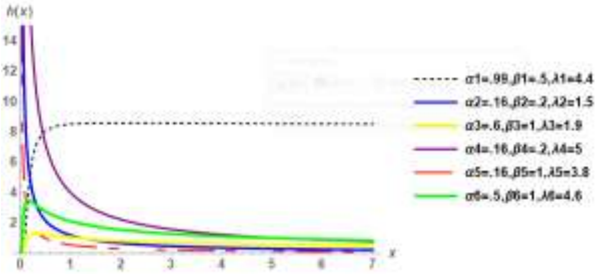


Figure 3 Figure 4

The plots of the probability density functions at different values of the parameters



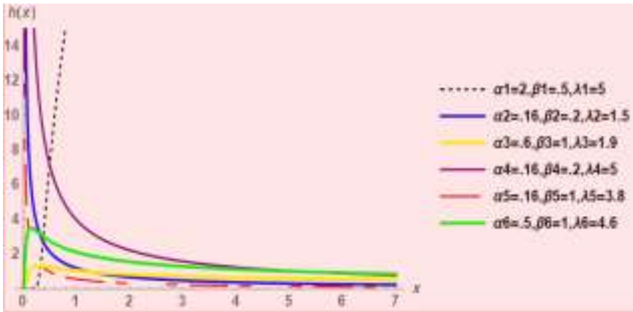


Figure 5 Figure 6

The plots of the hazard rate function at different values of the parameters

From Figure 1-5 displays OGE-IW $(\underline{\Psi})$ pdf for selected values of the parameters, where one can observe that the pdf of OGE-IW $(\underline{\Psi})$ distribution can be decreasing, unimodal or decreasing unimodal, and skewed to right (positive skewed). Also, from Figure 7-8 one can see that the plots of the hrf are a monotone decreasing, positive skewed and constant**.** This fact implies that the OGE-IW $(\underline{\Psi})$distribution is a flexible reliability mode and very useful for fitting data sets with various shapes. [For more details, see, Hassan *et al*. (2018)].

3. Bayesian Estimation

In this section, the Bayes estimators and CIs for the parameters of the OGE-IW $(\underline{\Psi})$ distribution can be obtained based on Type-II censored sample.

Suppose that $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(r)}$ is a Type II censored sample of size r obtained from a life-test on n items whose lifetimes have an OGE-IW $(\underline{\Psi})$ distribution. Then the *likelihood function* (LF) in this case is given by (6) that is

$$L(\underline{\Psi};\underline{x}) = C\{\prod_{i=1}^{r} f(x_{(i)};\underline{\Psi})\}[s(x_{(r)};\underline{\Psi})]^{n-r}, \tag{6}$$

where

$$C = \frac{n!}{(n-r)!}, \tag{7}$$

$\underline{\Psi} = (\alpha,\beta,\zeta)'$ and $f(x_{(i)};\ \underline{\Psi})$, $s(x_{(r)};\ \underline{\Psi})$ are respectively given by (1) and (3).

By substituting (1) and (3) into (6) yields, the LF of OGE-IW $(\underline{\Psi})$ distribution is given by

$$L(\underline{\Psi};\underline{x}) \propto \left((\alpha\beta\zeta)^r \prod_{i=1}^{r}\left[x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}\left(1-e^{-\beta x_i^{-\alpha}}\right)^{-2}\exp-\left(\frac{\zeta}{e^{\beta x_i^{-\alpha}}-1}\right)\right]\right)$$

$$\times\left[\exp-\left(\frac{\zeta}{e^{\beta x_r^{-\alpha}}-1}\right)\right]^{n-r}, \tag{8}$$

The natural logarithm of $L(\underline{\Psi};\underline{x})$ is given by

$$\ell \propto r\ln(\alpha)+r\ln(\beta)+r\ln(\zeta)-(\alpha+1)\sum_{i=1}^{r}\ln x_i-\beta\sum_{i=1}^{r}x_i^{-\alpha}-\sum_{i=1}^{r}\left(\frac{\zeta}{e^{\beta x_i^{-\alpha}}-1}\right)$$

$$-2\sum_{i=1}^{r} ln\left(1-e^{-\beta x_i^{-\alpha}}\right)-(n-r)ln\left(\frac{\zeta}{e^{\beta x_r^{-\alpha}}-1}\right) \tag{9}$$

The *maximum likelihood* (ML) estimators of $\underline{\Psi}$ can be obtained by differentiating ℓ in (9) with respect to α,β and ζ and then setting to zeros. Hence

$$\frac{\partial\ell}{\partial\zeta}=\frac{r}{\hat{\zeta}}-\sum_{i=1}^{r}\frac{1}{\left(e^{\hat{\beta}x_i^{-\hat{\alpha}}}-1\right)}-(n-r)\left(\frac{1}{\hat{\zeta}}\right)=0\,, \tag{10}$$

$$\frac{\partial\ell}{\partial\alpha}=\frac{r}{\hat{\alpha}}+\sum_{i=1}^{r}\ln x_i+\hat{\beta}\sum_{i=1}^{r}x_i^{-\hat{\alpha}}\ln x_i-\sum_{i=1}^{r}\left(\frac{\hat{\zeta}\hat{\beta}x_i^{-\hat{\alpha}}\ln x_i e^{\hat{\beta}x_i^{-\hat{\alpha}}}}{\left(e^{\hat{\beta}x_i^{-\hat{\alpha}}}-1\right)^2}\right)$$

$$-2\sum_{i=1}^{r}\left(\frac{\hat{\beta}x_i^{-\hat{\alpha}}\ln x_i e^{-\hat{\beta}x_i^{-\hat{\alpha}}}}{\left(1-e^{-\hat{\beta}x_i^{-\hat{\alpha}}}\right)}\right)+(n-r)\left[\frac{\left(e^{\hat{\beta}x_r^{-\hat{\alpha}}}-1\right)\hat{\beta}x_r^{-\hat{\alpha}}\ln x_r e^{\hat{\beta}x_r^{-\hat{\alpha}}}}{\hat{\zeta}}\right]=0, \tag{11}$$

and

$$\frac{\partial\ell}{\partial\beta}=\frac{r}{\hat{\beta}}-\sum_{i=1}^{r}x_i^{-\hat{\alpha}}+\sum_{i=1}^{r}\left(\frac{\hat{\zeta}x_i^{-\hat{\alpha}}e^{\hat{\beta}x_i^{-\hat{\alpha}}}}{\left(e^{\hat{\beta}x_i^{-\hat{\alpha}}}-1\right)^2}\right)-2\sum_{i=1}^{r}\left(\frac{x_i^{-\hat{\alpha}}e^{-\hat{\beta}x_i^{-\hat{\alpha}}}}{\left(1-e^{-\hat{\beta}x_i^{-\hat{\alpha}}}\right)}\right)$$

$$-(n-r)\left[\frac{\left(e^{\hat{\beta}x_r^{-\hat{\alpha}}}-1\right)x_r^{-\hat{\alpha}}e^{\hat{\beta}x_r^{-\hat{\alpha}}}}{\hat{\zeta}}\right]=0, \quad (12)$$

The ML estimators are obtained by equating the derivatives (10)-(12) to zeros. The system of non-linear can be solved numerically using Newton-Raphson method, to obtain the ML estimates of the parameters α, β and ζ.

Considering the prior knowledge of the vector of parameters $\underline{\Psi}=(\alpha,\beta,\zeta)'$, is adequately represented by gamma priors are assumed as independent prior distributions for the parameters Then the joint prior distribution of all the unknown parameters has a joint pdf given by

$$\pi(\underline{\Psi})=\prod_{j=1}^{3}\pi\left(\Psi_j\right), \quad (13)$$

where

$\Psi_j \sim$ gamma (a_j, b_j) and a_j, b_j are the hyper-parameters of the prior distribution for $j=1,2,3,$ with the following pdf

$$\pi(\underline{\Psi}_j;a_j,b_j)=\frac{b_j^{a_j}}{\Gamma(a_j)}\Psi_j^{a_j-1}\exp(-b_j\Psi_j),\underline{\Psi}_j>\underline{0};(a_j,b_j)>\underline{0}, \qquad j=1,2,3, \quad (14)$$

where $\Psi_1=\alpha, \Psi_2=\beta$ and $\Psi_3=\zeta$, $\Psi_j \sim$ gamma $(a_j,b_j), \underline{\Psi}=(\alpha,\beta,\zeta)'$. Then the joint prior distribution of all the unknown parameters has a joint pdf given by

$$\pi(\underline{\Psi};\underline{a},\underline{b}) \propto (\alpha^{a_1-1}\beta^{a_2-1}\zeta^{a_3-1})\exp[-(b_1\alpha+b_2\beta+b_3\zeta)], \qquad \underline{\Psi}>\underline{0};\ (\underline{a},\underline{b})>\underline{0}. \quad (15)$$

Combining the LF in (8) can be written as follows:

$$L(\underline{\Psi}|\underline{x}) \propto (\alpha\beta\zeta)^r \exp\left[-\alpha\sum_{i=1}^{r}\ln(x_{(i)})-\beta\sum_{i=1}^{r}x_i^{-\alpha}\right]$$
$$\times\exp\left[(n-r)ln\left(\frac{\zeta}{e^{\beta x_r^{-\alpha}}-1}\right)-\sum_{i=1}^{r}\left(\frac{\zeta}{e^{\beta x_i^{-\alpha}}-1}\right)-2\sum_{i=1}^{r}ln\left(1-e^{-\beta x_i^{-\alpha}}\right)\right] \quad (16)$$

and the joint prior distribution given by (15), then the joint posterior distribution of the parameters, for $\underline{\Psi}=(\alpha,\beta,\zeta)'$ can be obtained as follows:

$$\pi(\underline{\Psi}|\underline{x}) \propto L(\underline{\Psi}|\underline{x})\pi(\underline{\Psi};\underline{a},\underline{b}),$$
$$\propto (\alpha^{a_1+r-1}\beta^{a_2+r-1}\zeta^{a_3+r-1})\exp\left[-\alpha\sum_{i=1}^{r}\ln(x_{(i)})-\beta\sum_{i=1}^{r}x_i^{-\alpha}\right]$$
$$\times\exp\left[-\sum_{i=1}^{r}\left(\frac{\zeta}{e^{\beta x_i^{-\alpha}}-1}\right)-2\sum_{i=1}^{r}ln\left(1-e^{-\beta x_i^{-\alpha}}\right)\right]$$
$$\times\exp\left[(n-r)ln\left(\frac{\zeta}{e^{\beta x_r^{-\alpha}}-1}\right)-(b_1\alpha+b_2\beta+b_3\zeta)\right], \quad (17)$$

The joint posterior distribution given by (17) can be written as follows:

$$\pi(\underline{\Psi}|\underline{x})=K_1L(\underline{\Psi}|\underline{x})\pi(\underline{\Psi};\underline{a},\underline{b}),$$
$$=K_1\,(\alpha^{a_1+r-1}\beta^{a_2+r-1}\zeta^{a_3+r-1})\exp\left[-\alpha\sum_{i=1}^{r}\ln(x_{(i)})-\beta\sum_{i=1}^{r}x_i^{-\alpha}\right]$$

$$\times \exp\left[-\sum_{i=1}^{r}\left(\frac{\zeta}{e^{\beta x_i^{-\alpha}}-1}\right) - 2\sum_{i=1}^{r} ln\left(1-e^{-\beta x_i^{-\alpha}}\right)\right]$$

$$\times \exp\left[(n-r)ln\left(\frac{\zeta}{e^{\beta x_r^{-\alpha}}-1}\right) - (b_1\,\alpha + b_2\,\beta + b_3\,\zeta)\right], \tag{18}$$

where K_1 is a normalizing constant.
Then

$$K_1^{-1} = \int_{\underline{\Psi}} (\alpha^{a_1+r-1}\beta^{a_2+r-1}\zeta^{a_3+r-1}) \exp\left[-\alpha\sum_{i=1}^{r}\ln(x_{(i)}) - \beta\sum_{i=1}^{r} x_i^{-\alpha}\right]$$

$$\times \exp\left[-\sum_{i=1}^{r}\left(\frac{\zeta}{e^{\beta x_i^{-\alpha}}-1}\right) - 2\sum_{i=1}^{r} ln\left(1-e^{-\beta x_i^{-\alpha}}\right)\right]$$

$$\times \exp\left[(n-r)ln\left(\frac{\zeta}{e^{\beta x_r^{-\alpha}}-1}\right) - (b_1\,\alpha + b_2\,\beta + b_3\,\zeta)\right] d\underline{\Psi}, \tag{19}$$

where

$$\int_{\underline{\Psi}} = \int_{\alpha}\int_{\beta}\int_{\zeta}, d\underline{\Psi} = d\alpha\; d\beta\; d\zeta, \text{ and } \underline{\Psi} = (\alpha,\beta,\zeta). \tag{20}$$

The marginal posterior distributions of the parameters, $\underline{\Psi} = (\alpha,\beta,\zeta)$ are obtained from (21) as follows:

$$\pi\left(\Psi_\tau|\underline{x}\right) = \int_{\underline{\Psi}_j} \pi\left(\underline{\Psi}|\underline{x}\right) d\underline{\Psi}_j, \qquad \tau \neq j, \quad \tau, j = 1,2,3. \tag{21}$$

The Bayes estimators for the parameters of the OGE-IW$(\underline{\Psi})$ distribution are considered under the BLF. The estimator of a function using BLF is a mixture of the ML estimator, least squares estimators or any other estimator and the Bayes estimator using any loss function.

3.1 Bayesian estimation under balanced loss functions

Ahmadi *et al.* (2009) suggested the use of the *balanced loss function* (BLF), which was originated by Zellner (1994), to be of the form

$$L^*\left(\Psi,\tilde{\Psi}\right) = \omega\; l\left(\Psi,\hat{\Psi}\right) + (1-\omega)\; l\left(\Psi,\tilde{\Psi}\right), \tag{22}$$

where $l\left(\Psi,\tilde{\Psi}\right)$ is an arbitrary loss function, $\hat{\Psi}$ is a chosen target estimator of Ψ and the weight $\omega\epsilon[0, 1]$.

The BLF specializes to various choices of loss functions as a symmetric loss function and as an asymmetric loss function such as the absolute error loss, entropy,
linear exponential (LINEX) and *squared error loss* (SEL) and generalizes SEL functions.
The Bayes estimator of Ψ, using the BSEL function is given by

$$\tilde{\Psi}_{\mathrm{BSE}} = \omega\;\hat{\Psi}_{\mathrm{ML}} + (1-\omega)\;\tilde{\Psi}_{\mathrm{SE}}, \tag{23}$$

where $\widehat{\Psi}_{\text{ML}}$ is the ML estimator of Ψ and $\widetilde{\Psi}_{\text{SE}}$ is its Bayes estimator using SEL function. Also, the Bayes estimator using the BLL function of Ψ is obtained as follows:

$$\widetilde{\Psi}_{BL} = \frac{-1}{v} \ln\{\omega\, exp(-v\widehat{\Psi}_{ML}) + (1-\omega)\, E(exp(-v\Psi)\,|\underline{x})\}, \qquad (24)$$

where $v \neq 0$ is the shape parameter of BLL function.

Many authors used the symmetric and asymmetric distributions to construct Bayes estimators for various other distributions BLF. To learn more, [see Deniz (2006), AL-Hussaini and Hussein (2012), Abushal and AL-Zaydi (2017)].

The Bayes estimators are considered under the BSEL function; as a symmetric loss function and BLL function; as an asymmetric loss function.

3.1.1 Bayes estimators under BSEL function

From (18) and (23) the Bayes estimators of the parameters under BSEL can be obtained, respectively as follows

$$\widetilde{\Psi}_{\text{jBSE}} = \omega\widehat{\Psi}_{\text{jML}} + (1-\omega)E(\Psi_j|\underline{x})$$

$$= \omega\widehat{\Psi}_{\text{jML}} + (1-\omega)\int_{\underline{\Psi}} \Psi_j\, \pi(\underline{\Psi}|\underline{x})d\underline{\Psi}$$

$$= \omega\widehat{\Psi}_{jML} + (1-\omega)K_1\int_{\underline{\Psi}} \Psi_j\,(\alpha^{a_1+r-1}\beta^{a_2+r-1}\zeta^{a_3+r-1})\exp[-\alpha\textstyle\sum_{i=1}^{r}\ln(x_{(i)}) - \beta\sum_{i=1}^{r} x_i^{-\alpha}]$$

$$\times \exp\left[-\sum_{i=1}^{r}\left(\frac{\zeta}{e^{\beta x_i^{-\alpha}}-1}\right) - 2\sum_{i=1}^{r} ln\left(1-e^{-\beta x_i^{-\alpha}}\right)\right]$$

$$\times \exp\left[(n-r)ln\left(\frac{\zeta}{e^{\beta x_r^{-\alpha}}-1}\right) - (b_1\,\alpha + b_2\,\beta + b_3\,\zeta)\right]d\underline{\Psi}, \qquad \Psi_j > 0, j = 1,2,3 \qquad (25)$$

where $\Psi_1 = \alpha, \Psi_2 = \beta$ and $\Psi_3 = \zeta$, $\widehat{\Psi}_{jML}$ is the estimator of Ψ_j using the ML method based on (10)-(11) and (12) , K_1^{-1} is given by (19), $\int_{\Psi}$ and $d\underline{\Psi}$ are given by (20).

The Bayes estimators for the unknown parameters, $\underline{\Psi} = (\alpha,\beta,\zeta)'$, using BSEL function should be solved numerically by substituting Ψ_j by α or β or ζ in (25).

3.1.2 Bayes estimators BLL function

From (18) and (24) the Bayes estimators of the parameters under BLL can be calculated, respectively as follows

$$\widetilde{\Psi}_{j\,BBL} = \frac{-1}{v}\ln\left\{\omega\, exp(-v\widehat{\Psi}_{ML}) + (1-\omega)\int_{\underline{\Psi}} exp(-v\Psi)\,\pi(\underline{\Psi}|\underline{x})d\underline{\Psi}\right\}$$

$$\widetilde{\Psi}_{j\,BBL} = \frac{-1}{v}\ln\{\omega\exp(-v\widehat{\Psi}_{jML}) + (1-\omega)K_1\int_{\underline{\Psi}}(\alpha^{a_1+r-1}\beta^{a_2+r-1}\zeta^{a_3+r-1})$$

$$\times \exp\left[-\left(\upsilon\Psi_j + b_1\,\alpha + b_2\,\beta + b_3\,\zeta + \alpha\sum_{i=1}^{r}\ln\left(x_{(i)}\right) + \beta\sum_{i=1}^{r}x_i^{-\alpha}\right)\right]$$

$$\times \exp\left[-\sum_{i=1}^{r}\left(\frac{\zeta}{e^{\beta x_i^{-\alpha}}-1}\right) - 2\sum_{i=1}^{r}ln\left(1-e^{-\beta x_i^{-\alpha}}\right) + (n-r)ln\left(\frac{\zeta}{e^{\beta x_r^{-\alpha}}-1}\right)\right]d\underline{\Psi}\},$$

$$\Psi_j > 0, j = 1,2,3 \quad (26)$$

where $\Psi_1 = \alpha, \Psi_2 = \beta$ and $\Psi_3 = \zeta$, $\hat{\Psi}_{jML}$ is the estimator of Ψ_j using the ML method based on (10)-(11) and (12) , $K_1^{-1}$is given by (19), $\int \square_{\Psi}$ and $d\underline{\Psi}$ are given by (20).

The Bayes estimators for the unknown parameters, $\underline{\Psi} = (\alpha, \beta, \zeta)'$, using BLL function may be computed by substituting Ψ_j by α or β or ζ in (26).

4. E-Bayesian Estimation

The E-Bayes estimators of the parameters of the OGE-IW$(\underline{\Psi})$ distribution based on Type-II censored sample are considered in this subsection.

According to Han (2007), the hyper-parameters a_j and b_j ought to be chosen to provide that $\pi(\theta_j; a_j, b_j)$, are decreasing functions of Ψ_j , ($j = 1,2,3$).

The derivative of $\pi(\underline{\Psi}_j; a_j, b_j)$ with respect to $\underline{\Psi}_j$ is given below

$$\frac{d\,\pi(\underline{\Psi}_j; a_j, b_j)}{d\underline{\Psi}_j} = \frac{b_j^{a_j}}{\Gamma(a_j)}\Psi_j^{a_j-2}\,exp\left(-b_j\Psi_j\right)\left[\left(a_j-1\right)-b_j\Psi_j\right], \qquad j = 1,2,3, \quad (27)$$

for $0 < a_j < 1$ and $b_j > 0$, then $\frac{d\,\pi(\underline{\Psi}_j; a_j, b_j)}{d\underline{\Psi}_j} < 0$, which means that $\pi(\underline{\Psi}_j; a_j, b_j)$ can be decreasing functions of $\underline{\Psi}_j$.

To derive the E-Bayes estimators of the parameters, three alternative distributions for the hyper-parameters a_j and b_j. These distributions are used to investigate the effect of different prior distributions on the E-Bayesian estimation of $\underline{\Psi}_j$.

Assuming that hyper-parameters a_j and b_j are independent, it may be deduced that they have bivariate density functions.

$$\pi_\hbar\left(a_j, b_j\right) = \pi_\hbar\left(a_j\right)\pi_\hbar\left(b_j\right), \qquad j = 1,2,3 \;\; \hbar = 1,2,\ldots,6. \quad (28)$$

Then, the bivariate uniform hyper-prior distributions are provided as follows:

$$\pi_{\hbar}\left(a_{j}, b_{j}\right)=\frac{2\left(c_{j}-b_{j}\right)}{c_{j}^{2}}, \qquad 0<a_{j}<1, 0<b_{j}<c_{j}, \tag{29}$$

$$\pi_{\hbar}\left(a_{j}, b_{j}\right)=\frac{1}{c_{j}}, \qquad 0<a_{j}<1,\ 0<b_{j}<c_{j}, \tag{30}$$

$$\pi_{\hbar}\left(a_{j}, b_{j}\right)=\frac{2 b_{j}}{c_{j}^{2}}, \qquad 0<a_{j}<1,\ 0<b_{j}<c_{j}. \tag{31}$$

The E-Bayes estimators of $\underline{\Psi}_{j}$ (expectation of the Bayes estimators of $\underline{\Psi}_{j}$) could be obtained as follows:

$$\underline{\widetilde{\Psi}}_{jEB}=E_{\pi_{\hbar}}\left(\widetilde{\Psi}_{jB}\left(a_{j}, b_{j}\right)\right)=\iint_{D} \underline{\widetilde{\Psi}}_{jB}\left(a_{j}, b_{j}\right) \pi_{\hbar}\left(a_{j}, b_{j}\right)\ d a_{j}\ d b_{j},$$
$$j=1,2,3, \hbar=1,2, \ldots, 6, \tag{32}$$

where $E_{\pi_{\hbar}}(\hbar=1,2, \ldots, 6)$ indicates for the expectation of the bivariate hyper prior distributions, D is the domain of the function $\pi_{\hbar}\left(a_{j}, b_{j}\right)$ and $\underline{\widetilde{\Psi}}_{jB}\left(a_{j}, b_{j}\right)$ are the Bayes estimators of the parameters $\underline{\Psi}_{j}, j=1,2,3$ based on BSEL and BLL functions.

The E-Bayes estimators of the parameters of the OGE-IW$(\underline{\Psi})$ distribution, which are based on a Type-II censored sample, are examined under two distinct loss functions: the asymmetric BLL function and the symmetric BSEL function.

4.1 E-Bayes estimators under BSEL function

The three E-Bayes estimators of the parameters Ψ_j under BSEL function can be obtained by substituting (28) and (47)-(49) in (50) as indicated, respectively, below

$$\widetilde{\Psi}_{j\, EBBSES_{1}}=\frac{2}{c_{j}^{2}} \int_{0}^{1} \int_{0}^{c_{j}}\left\{\omega \widehat{\Psi}_{jML}+(1-\omega) K_{1} \int_{\Psi} \Psi_{j}\left(\alpha^{a_{1}+r-1} \beta^{a_{2}+r-1} \zeta^{a_{3}+r-1}\right)\right.$$
$$\times \exp \left[-\alpha \sum_{i=1}^{r} \ln \left(x_{(i)}\right)-\beta \sum_{i=1}^{r} x_{i}^{-\alpha}-\sum_{i=1}^{r}\left(\frac{\zeta}{e^{\beta x_{i}^{-\alpha}}-1}\right)-2 \sum_{i=1}^{r} \ln \left(1-e^{-\beta x_{i}^{-\alpha}}\right)\right]$$
$$\left.\times \exp \left[(n-r) \ln \left(\frac{\zeta}{e^{\beta x_{r}^{-\alpha}}-1}\right)-\left(b_{1} \alpha+b_{2} \beta+b_{3} \zeta\right)\right] d \underline{\Psi}\right\} \times\left(c_{j}-b_{j}\right) d b_{j} d a_{j}, \tag{33}$$

$$\widetilde{\Psi}_{j\, EBBSES_{2}}=\frac{1}{c_{j}} \int_{0}^{1} \int_{0}^{c_{j}}\left\{\omega \widehat{\Psi}_{jML}+(1-\omega) K_{1} \int_{\underline{\Psi}} \Psi_{j}\left(\alpha^{a_{1}+r-1} \beta^{a_{2}+r-1} \zeta^{a_{3}+r-1}\right)\right.$$
$$\times \exp \left[-\alpha \sum_{i=1}^{r} \ln \left(x_{(i)}\right)-\beta \sum_{i=1}^{r} x_{i}^{-\alpha}-\sum_{i=1}^{r}\left(\frac{\zeta}{e^{\beta x_{i}^{-\alpha}}-1}\right)-2 \sum_{i=1}^{r} \ln \left(1-e^{-\beta x_{i}^{-\alpha}}\right)\right]$$
$$\left.\times \exp \left[(n-r) \ln \left(\frac{\zeta}{e^{\beta x_{r}^{-\alpha}}-1}\right)-\left(b_{1} \alpha+b_{2} \beta+b_{3} \zeta\right)\right] d \underline{\Psi}\right\} d b_{j} d a_{j}, \tag{34}$$

and

$$\tilde{\Psi}_{j\,EBBSES_3} = \frac{2}{c_j^2}\int_0^1\int_0^{c_j} b_j\left\{\omega\hat{\Psi}_{jML} + (1-\omega)K_1\int_{\Psi}\Psi_j\left(\alpha^{a_1+r-1}\beta^{a_2+r-1}\zeta^{a_3+r-1}\right)\right.$$

$$\times \exp\left[-\alpha\sum_{i=1}^{r}\ln\left(x_{(i)}\right) - \beta\sum_{i=1}^{r}x_i^{-\alpha} - \sum_{i=1}^{r}\left(\frac{\zeta}{e^{\beta x_i^{-\alpha}}-1}\right) - 2\sum_{i=1}^{r}ln\left(1-e^{-\beta x_i^{-\alpha}}\right)\right]$$

$$\left.\times \exp\left[(n-r)ln\left(\frac{\zeta}{e^{\beta x_r^{-\alpha}}-1}\right) - \left(b_1\,\alpha + b_2\,\beta + b_3\,\zeta\right)\right] d\underline{\Psi}\right\} db_j da_j, \qquad (35)$$

where $j = 1,2,3$, $\Psi_1 = \alpha, \Psi_2 = \beta$ and $\Psi_3 = \zeta$, $\hat{\Psi}_{jML}$ is the estimator of Ψ_j using the ML approach based on (10)-(12) and K_1^{-1} is defined as in (19), $\int_{\Psi}$ and $d\underline{\Psi}$ are given by (20).

The three E-Bayes estimators of each of the parameters α, β and ζ under BSEL function can be computed by replacing with $j = 1, j = 2$ and $j = 3$ in (33)-(35), respectively.

4.2 E-Bayes estimators BLL function

The three E-Bayes estimators of the parameters Ψ_j using the BLL function are calculated by substituting (26) and (29)-(31) in (32)

$$\tilde{\Psi}_{j\,EBBL_1} = \frac{2}{c_j^2}\int_0^1\int_0^{c_j}\left\{\frac{-1}{\upsilon}\ln\,\omega\exp\left(-\upsilon\hat{\Psi}_{jML}\right) + (1-\omega)K_1\int_{\underline{\Psi}}\left(\alpha^{a_1+r-1}\beta^{a_2+r-1}\zeta^{a_3+r-1}\right)\right.$$

$$\times \exp\left[-\left(\upsilon\Psi_j + b_1\,\alpha + b_2\,\beta + b_3\,\zeta + \alpha\sum_{i=1}^{r}\ln\left(x_{(i)}\right) + \beta\sum_{i=1}^{r}x_i^{-\alpha}\right)\right]$$

$$\left.\times \exp\left[-\sum_{i=1}^{r}\left(\frac{\zeta}{e^{\beta x_i^{-\alpha}}-1}\right) - 2\sum_{i=1}^{r}ln\left(1-e^{-\beta x_i^{-\alpha}}\right) + (n-r)ln\left(\frac{\zeta}{e^{\beta x_r^{-\alpha}}-1}\right)\right]d\underline{\Psi}\right\}$$

$$\times\left(c_j - b_j\right) db_j da_j, \qquad (36)$$

$$\tilde{\Psi}_{j\,EBBL_2} = \frac{1}{c_j}\int_0^1\int_0^{c_j}\left\{\frac{-1}{\upsilon}\ln\,\omega\exp\left(-\upsilon\hat{\Psi}_{jML}\right) + (1-\omega)K_1\int_{\underline{\Psi}}\left(\alpha^{a_1+r-1}\beta^{a_2+r-1}\zeta^{a_3+r-1}\right)\right.$$

$$\times \exp\left[-\left(\upsilon\Psi_j + b_1\,\alpha + b_2\,\beta + b_3\,\zeta + \alpha\sum_{i=1}^{r}\ln\left(x_{(i)}\right) + \beta\sum_{i=1}^{r}x_i^{-\alpha}\right)\right]$$

$$\left.\times \exp\left[-\sum_{i=1}^{r}\left(\frac{\zeta}{e^{\beta x_i^{-\alpha}}-1}\right) - 2\sum_{i=1}^{r}ln\left(1-e^{-\beta x_i^{-\alpha}}\right) + (n-r)ln\left(\frac{\zeta}{e^{\beta x_r^{-\alpha}}-1}\right)\right]d\underline{\Psi}\right\}db_j da_j, \qquad (37)$$

and

$$\tilde{\Psi}_{j\,EBBL_3} = \frac{2}{c_j^2}\int_0^1\int_0^{c_j} b_j\left\{\frac{-1}{\upsilon}\ln\,\omega\exp\left(-\upsilon\hat{\Psi}_{jML}\right) + (1-\omega)K_1\int_{\underline{\Psi}}\left(\alpha^{a_1+r-1}\beta^{a_2+r-1}\zeta^{a_3+r-1}\right)\right.$$

$$\times \exp\left[-\left(\upsilon\Psi_j + b_1\,\alpha + b_2\,\beta + b_3\,\zeta + \alpha\sum_{i=1}^{r}\ln\left(x_{(i)}\right) + \beta\sum_{i=1}^{r}x_i^{-\alpha}\right)\right]$$

$$\times \exp\left[-\sum_{i=1}^{r}\left(\frac{\zeta}{e^{\beta x_i^{-\alpha}}-1}\right)-2\sum_{i=1}^{r} ln\left(1-e^{-\beta x_i^{-\alpha}}\right)+(n-r)ln\left(\frac{\zeta}{e^{\beta x_r^{-\alpha}}-1}\right)\right]d\underline{\Psi}\}db_j da_j,$$

(38)

where $j=1,2,3,\ \Psi_1=\alpha, \Psi_2=\beta$ and $\Psi_3=\zeta$, $\hat{\Psi}_{jML}$ is the estimator of Ψ_j using the ML method based on (10)-(11) and (12) , $K_1^{-1}$is given by (19), $\int_{\Psi}$ and $d\underline{\Psi}$ are given by (20).

To derive the three E-Bayes estimators of each of the parameters α, β and ζ under BLL function, substitute $j=1, j=2$ and $j=3$ in (36)-(38), correspondingly.

5. Numerical Illustration

This section uses both simulated and real datasets to examine the theoretical outcomes of Bayes and E-Bayes estimations for accuracy.

5.1 Simulation algorithm

A simulation analysis is carried out in this subsection using data obtained from the OGE-IW$(\underline{\Psi})$ distribution to demonstrate the performance of the presented Bayes and E-Bayes estimates. Calculated are the Bayes and E- Bayes estimates of the parameters based on the Type-II censoring sample. In addition, CIs are computed for the parameters. With the R programming language, all simulation investigations are carried out.

The steps in the simulation procedure using Type-II censored data are as follows:

Step 1: random samples of size n are produced from the OGE-IW$(\underline{\Psi})$ distribution for given values of α, β and ζ.

- According to Hassan *et al.* (2018), the uniform distribution can be transformed into the OGEIW$(\underline{\Psi})$ distribution as follows:

$$x_u=\left[\frac{1}{\beta} ln\left[\frac{-\lambda}{ln(1-u)}+1\right]\right]^{\frac{-1}{\alpha}},\ \ 0<u<1.$$

Step 2: Generate a_j and b_j from the bivariate uniform hyperprior distributions; $\pi_\hbar(a_j, b_j)$,

$j=1,2,3, \hbar=1,2,\ldots,9,$ given in (47)-(49).

Step 3: For given values of a_j and b_j, generate α, β and ζ from the gamma prior distributions.

Step 4: For each sample size n, sort the x_i`s, such that $x_{(1)} \le x_{(2)} \le \cdots \le x_{(n)}$.

Step 5: The number of failures r are chosen to be less than or equal the sample size n.

Step 6: Calculate the Bayes and E-Bayes estimates for the parameters using the BSEL and BLL functions.

Step 7: Repeat all the previous steps N=10000 times for the samples of size n = 30, 60, and 100.

Their performance is assessed based on how accurate the estimates are. It is convenient to use the estimated risk $(ER) = \frac{\sum_{i=1}^{N}(\text{estimatedvalue}-\text{truevalue})^2}{N}$ to investigate the precision and variation of the estimates.

Table 1-4 display the Bayes and E-Bayes averages and ERs of the unknown parameter of the OGE-IW$(\underline{\Psi})$ distribution under BSEL and BLL functions using Type-II censored data, where the censoring level is 60% , 80% and 100% n for each sample size, and the specified values of parameters are (Case 1: $\alpha = 2.3, \beta = 1.5$ and $\zeta = 1.1$) and (Case 2: $\alpha = .3, \beta = .5$ and $\zeta = .6$).
Tables 5-6 display the Bayes and E-Bayes averages and *expected risks* (ERs) for the unknown parameters under BSEL and BLL functions based on Type-II censoring for different weights ω; $\omega = 0,0.4,0.7$ and 1 where when $\omega = 0$, the Bayes estimates using the BSEL or BLL functions are obtained and when $\omega = 1$, the ML estimates are obtained.

5.2 Some applications

Showing how the suggested methods can be used in practice is the main goal of this subsection. Utilizing three real-world datasets, this is accomplished. Utilizing the R programming language, the Kolmogorov-Smirnov goodness-of-fit test is used to show that the OGE-IW$(\underline{\Psi})$ distribution is fitted to the three real datasets.

Application 1

The first data introduced by Liu *et al.* (2021). The data refer to the survival times of patients suffering from the COVID-19 epidemic in China. The considered data set representing the survival times of patients from the time admitted to the hospital until death. Among them, a group of fifty-three (53) COVID-19 patients were found in critical condition in hospital from January to February 2020. The data set can be retrieved from https://www.worldometers.info/coronavirus/ and is given by: 0.054, 0.064, 0.704, 0.816, 0.235, 0.976, 0.865, 0.364, 0.479, 0.568, 0.352, 0.978, 0.787, 0.976, 0.087, 0.548, 0.796, 0.458, 0.087, 0.437, 0.421, 1.978, 1.756, 2.089, 2.643, 2.869, 3.867, 3.890, 3.543, 3.079, 3.646, 3.348, 4.093, 4.092, 4.190, 4.237, 5.028, 5.083, 6.174, 6.743, 7.274, 7.058, 8.273, 9.324, 10.827, 11.282, 13.324, 14.278, 15.287, 16.978, 17.209, 19.092 and 20.083.

Application 2

The second data is given by Murthy *et al.* (2004). The data refers to the time between failures for a repairable item: 1.43, 0.11, 0.71, 0.77, 2.63, 1.49, 3.46, 2.46, 0.59, 0.74, 1.23, 0.94, 4.36, 0.40, 1.74, 4.73, 2.23, 0.45, 0.70, 1.06, 1.46, 0.30, 1.82, 2.37, 0.63, 1.23, 1.24, 1.97, 1.86 and 1.17.

Application 3

The third data set is given by Singh and Maddala (1976) the data represents the strength of 1.5 cm glass fibers for 60 devices.

The data set is:0.636, 0.252, 0.157, 0.187, 2.771, 0.209, 0.617, 2.078, 1.013, 0.499, 0.431, 0.642, 0.46, 0.749,0.205, 0.576, 0.439, 0.471, 0.262, 0.387, 0.324, 0.424, 0.548, 1.794, 1.233, 0.915, 0.702, 0.417,0.337, 0.435, 0.359, 0.293, 0.147, 0.87, 0.608, 0.153, 0.098, 0.557, 0.415, 0.122, 0.912, 0.341,0.725, 0.364, 0.24, 0.594, 0.325, 0.416, 0.08, 0.582, 1.257, 1.575, 0.48, 0.909, 0.17, 0.319, 0.09,0.154, 2.248 and 0.292.

The Kolmogorov–Smirnov goodness of fit test is applied to check the validity of the fitted model. The p- values are given, respectively, 0.9746, 0.9578 and 0.9867. The p value given in each case showed that the model fits the data very well. The Bayes and E-Bayes estimates and *standard errors* (SEs) of the unknown parameters for the real datasets using BSEL and BLL functions are presented in Tables 7- 10.

5.3 Concluding remarks

1) According to Tables 1–4, as the sample size increases, the precision of the ER improves.
2) Tables 5 and 6 show that an increase in weight decreases improves the ER's accuracy.
3) Since the ERs of the E-Bayes estimates of the parameters are always lower than the ERs of the Bayes estimates, the E-Bayes estimators consistently perform better than the Bayes estimators, as in Tables 1–6.
4) From Tables 1–6, the ERs of Bayes and E-Bayes estimates using the BLL function are lower than the ERs of Bayes and E-Bayes estimates using the BSEL function, indicating that the Bayes and E-Bayes estimators using the BLL function typically perform better than the Bayes and E-Bayes estimators using the BSEL function.
5) Tables 5 and 6 demonstrate that when $\omega = 0$, the Bayes estimates using the BSEL or BLL functions are produced, and when $\omega = 1$, the ML estimates are obtained.

6. General Conclusion

In this study, Type-II censored samples are used to derive the Bayes and E-Bayes estimators for the OGEIW$\left(\underline{\Psi}\right)$ distribution's parameters. The estimators are considered under two different loss functions, the BSEL function; as a symmetric loss function and BLL function; as an asymmetric loss function. The BLF is a mixture of Bayes and non-Bayes estimators. On the basis of informative gamma prior distributions, the estimators are derived. Furthermore, a simulation study and an application employing actual datasets are used to assess the performance of the suggested Bayes and E-Bayes estimations. Numerical calculations generally demonstrated that the accuracy of the ER improves with increasing sample size and decreasing weight ω. Because the ERs of the E-Bayes estimates of the parameters are always lower than the ERs of the Bayes estimates, the E-Bayes estimators consistently perform better than the Bayes estimators. Future distribution theory research could benefit from employing the E-Bayesian and Bayesian approaches for estimating the parameters of the OGEIW$\left(\underline{\Psi}\right)$ distribution using various loss functions, such as precautionary and general entropy loss functions.

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Table 1: Estimated risks of the OGEIW parameters under balanced squared error loss function based on Type-II censoring ($\boldsymbol{N = 10000, r = 0.6n, 0.8n \text{ and } 0.1n, \alpha = 2.3, \beta = 1.5, \zeta = 1.1, \omega = 0.3}$)

n	r	ER		ER		ER	
		$\tilde{\alpha}_B$	$\tilde{\alpha}_{EB}$	$\tilde{\beta}_B$	$\tilde{\beta}_{EB}$	$\tilde{\zeta}_B$	$\tilde{\zeta}_{EB}$
30	18	1.4670	0.1794	0.1180	0.0244	1.8096	0.0457
			0.0124		0.1714		0.0205
			0.0246		0.0830		0.1023
	24	1.0987	0.0341	1.1594	0.1005	1.4154	0.4932
			0.0491		0.1373		0.0120
			0.0256		0.0174		0.4455
	30	0.5761	0.2153	0.3719	0.0453	1.7683	0.0527
			0.1735		0.0172		0.1065
			0.1514		0.0396		0.0169
60	36	0.4148	0.0142	0.1616	0.0515	0.3897	0.0141
			0.4199		0.0152		0.0224
			0.0494		0.0872		0.3762
	48	0.3154	0.0202	0.0298	0.0400	0.2807	0.0232
			0.0263		0.1128		0.1028
			0.1059		0.0385		0.2065
	60	0.1709	0.0196	0.1206	0.0382	0.9089	0.0439
			0.0323		0.1326		0.0663
			0.0206		0.1528		0.0502
100	60	0.2220	0.0090	0.1599	0.0496	0.0633	0.0671
			0.0465		0.0121		0.2380
			0.0222		0.0246		0.0579
	80	0.2847	0.0814	0.0293	0.1464	0.0246	0.1405
			0.0178		0.1240		0.2758
			0.0319		0.0362		0.0102
	100	0.0330	0.2359	0.0728	0.2758	0.1593	0.0407
			0.0468		0.0350		0.0258
			0.0901		0.0131		0.8200
200	120	0.1460	0.1249	0.0719	0.0442	0.1764	0.1106
			0.1096		0.0947		0.6762
			0.0352		0.1349		0.0791
	160	0.0579	0.0199	0.0877	0.0416	0.0377	0.2113
			0.0422		0.0165		0.2863
			0.1097		0.2997		0.1062
	200	0.0281	0.0848	0.0676	0.1317	0.0887	0.0414
			0.0245		0.0658		0.0316
			0.4349		0.1118		0.0213

Table 2: Estimated risks of the OGEIW parameters under balanced linear exponential loss function based on Type-II censoring($N = 10000, r = 0.6$n, 0.8n and 0.1n$, \alpha = 2.3, \beta = 1.5, \zeta = 1.1, \omega = 0.3, \nu = -2$)

n	r	ER		ER		ER	
		$\tilde{\alpha}_B$	$\tilde{\alpha}_{EB}$	$\tilde{\beta}_B$	$\tilde{\beta}_{EB}$	$\tilde{\zeta}_B$	$\tilde{\zeta}_{EB}$
30	18	0.1191	0.0259	0.1804	0.0919	1.2190	0.0925
			0.0192		0.1272		0.1691
			0.1470		0.2978		0.0172
	24	0.0731	0.0187	0.0665	0.0281	0.8719	0.2994
			0.0377		0.0336		0.0116
			0.0406		0.0104		0.3626
	30	0.2130	0.1699	0.1010	0.0459	1.8136	0.0908
			0.0276		0.2477		0.1034
			0.1155		0.0153		0.0168
60	36	0.0979	0.0083	0.0938	0.0099	0.2696	0.1045
			0.0549		0.0341		0.0475
			0.1112		0.6944		0.1574
	48	0.0820	0.1019	0.0526	0.0526	0.0675	0.0062
			0.0077		0.5187		0.0087
			0.0903		0.5056		0.1086
	60	0.1036	0.1333	0.1711	0.0448	0.2446	0.1848
			0.0271		0.4251		0.0131
			0.2800		0.0269		0.0682
100	60	0.1113	0.0674	0.1703	0.2869	0.3657	0.0247
			0.2194		0.1383		0.1182
			0.1089		0.2879		0.2456
	80	0.1336	0.0454	0.0711	0.0681	0.0634	0.0280
			0.3875		0.0294		0.0055
			0.0131		0.0059		0.0101
	100	0.0430	0.0581	0.0341	0.0115	0.0976	0.0263
			0.0605		0.0371		0.1632
			0.0531		0.0270		0.0625
200	120	0.5069	0.1284	0.2865	0.0314	0.0819	0.0243
			0.0159		0.2296		0.0523
			0.0366		0.1480		0.0034
	160	0.0642	0.0831	0.6723	0.1310	0.8931	0.1355
			0.0276		0.0632		0.0930
			0.0782		0.0708		0.0055
	200	0.4449	0.0139	0.0813	0.0271	0.3867	0.0071
			0.0366		0.0226		0.4772
			0.1439		0.0385		0.0862

Table 3: Estimated risks of the OGEIW parameters under balanced squared error loss function based on Type-II censoring ($N = 10000, r = 0.6$n, 0.8n and 0.1n, $\alpha = 0.3, \beta = 0.5, \zeta = 0.6, \omega = 0.3$)

n	r	ER		ER		ER	
		$\tilde{\alpha}_B$	$\tilde{\alpha}_E$	$\tilde{\beta}_B$	$\tilde{\beta}_{EB}$	$\tilde{\zeta}_B$	$\tilde{\zeta}_{EB}$
30	18	0.1656	0.0462	0.3179	0.0211	0.3413	0.0745
			0.0279		0.0871		0.0921
			0.2334		0.1332		0.2431
	24	0.0955	0.0124	0.1126	0.0810	0.3412	0.2611
			0.2291		0.2040		0.0965
			0.0713		0.2630		0.2261
	30	0.0889	0.4526	0.1098	0.0165	0.3386	0.0558
			0.1773		0.0734		0.0877
			0.1815		0.0175		0.0755
60	36	0.0872	0.0479	0.0934	0.0493	0.3385	0.3118
			0.0168		0.0720		0.0289
			0.0290		0.1172		0.0788
	48	0.0845	0.0574	0.0589	0.0199	0.3381	0.0173
			0.0255		0.0683		0.0387
			0.0253		0.1563		0.2188
	60	0.0690	0.0699	0.0506	0.0124	0.3374	0.0377
			0.0918		0.1148		0.1697
			0.0604		0.0452		0.6018
100	60	0.0689	0.0349	0.0505	0.0126	0.3345	0.0146
			0.1539		0.1090		0.0306
			0.4535		0.0976		0.7604
	80	0.0588	0.0332	0.0490	0.0415	0.3268	0.2195
			0.0411		0.0860		0.0276
			0.2451		0.1159		0.1435
	100	0.0438	0.0707	0.0439	0.2036	0.3246	0.0126
			0.0964		0.0268		0.1439
			0.0588		0.1129		0.3912
200	120	0.0386	0.0779	0.0255	0.0697	0.3131	0.0842
			0.0970		0.0273		0.1924
			0.1255		0.2817		0.0473
	160	0.0200	0.0130	0.0202	0.0329	0.3047	0.0226
			0.0754		0.0255		0.1012
			0.0557		0.0647		0.3556
	200	0.0175	0.4527	0.0162	0.0224	0.3034	0.2018
			0.0247		0.0261		0.0336
			0.0519		0.4188		0.0730

Table 4: Estimated risks of the OGEIW parameters under balanced linear exponential loss function based on Type-II censoring ($N = 10000, r = 0.6\text{n}, 0.8\text{n}$ and $0.1\text{n}, \alpha = 0.3, \beta = 0.5, \zeta = 0.6, \omega = 0.3, \nu = -2$)

n	r	ER		ER		ER	
		$\tilde{\alpha}_B$	$\tilde{\alpha}_{EB}$	$\tilde{\beta}_B$	$\tilde{\beta}_{EB}$	$\tilde{\zeta}_B$	$\tilde{\zeta}_{EB}$
30	18	0.0750	0.0963	0.0825	0.0936	0.3447	0.0086
			0.0338		0.0730		0.0111
			0.1265		0.0168		0.0688
	24	0.0636	0.0063	0.0657	0.0098	0.3337	0.1054
			0.2182		0.0097		0.0098
			0.2113		0.0098		0.1460
	30	0.0549	0.0064	0.0653	0.0352	0.3335	0.0262
			0.0146		0.0205		0.0411
			0.0902		0.4367		0.4126
60	36	0.0544	0.0404	0.0488	0.0124	0.3302	0.6238
			0.0076		0.1282		0.0288
			0.0630		0.0243		0.0845
	48	0.0391	0.0491	0.0420	0.0855	0.3292	0.0170
			0.0810		0.0235		0.0868
			0.0418		0.2346		0.0905
	60	0.0363	0.0279	0.0245	0.0740	0.3288	0.0177
			0.2803		0.1643		0.0175
			0.0373		0.1252		0.2269
100	60	0.0358	0.0216	0.0207	0.0381	0.3284	0.0943
			0.0085		0.3305		0.1310
			0.0751		0.0237		0.0158
	80	0.0354	0.1355	0.0175	0.2269	0.3261	0.0493
			0.0066		0.0928		0.2600
			0.0393		0.0256		0.0058
	100	0.0265	0.0963	0.0149	0.2489	0.3246	0.0186
			0.0338		0.0255		0.0213
			0.1265		0.0226		0.2156
200	120	0.0123	0.2032	0.0147	0.1399	0.3179	0.0179
			0.0190		0.0149		0.0089
			0.0983		0.0105		0.1483
	160	0.0096	0.0275	0.0139	0.0383	0.3113	0.0171
			0.0377		0.0894		0.0249
			0.0246		0.0293		0.0136
	200	0.0066	0.0297	0.0134	0.1052	0.0412	0.1866
			0.0673		0.0158		0.0140
			0.1003		0.1644		0.0774

Table 5: Estimated risks of the OGEIW parameters under balanced squared error loss function based on Type-II censoring $(N = 10000, n = 60, r = 0.6\text{n}, \alpha = 0.8, \beta = 1.15, \zeta = 2.6)$

$\boldsymbol{\Psi}$	$\boldsymbol{\omega = 0}$		$\boldsymbol{\omega = 0,4}$		$\boldsymbol{\omega = 0,7}$		$\boldsymbol{\omega = 1}$	
$\boldsymbol{\alpha}$	0.1041	0.0283	0.1172	0.3590	0.1276	0.1135	0.2472	0.3590
		0.2114		0.0515		0.1203		0.3626
		0.2906		0.3626		0.0294		0.4862
$\boldsymbol{\beta}$	0.0220	0.0528	0.2031	0.1685	0.4981	0.0490	0.9610	0.1627
		0.0311		0.0484		0.0503		0.3419
		0.0433		0.0417		0.0265		0.4392
$\boldsymbol{\zeta}$	5.0166	0.2323	5.1431	0.0485	5.3200	0.1497	5.6180	0.2800
		0.0634		0.0291		0.0663		0.2415
		0.0120		0.0504		0.0216		0.2911

Table 6: Estimated risks of the OGEIW parameters under balanced inear exponential loss function based on Type-II censoring $(N = 10000, n = 60, r = 0.6\text{n}, \nu = -2, \alpha = 0.8, \beta = 1.15, \zeta = 2.6)$

$\boldsymbol{\Psi}$	$\omega = 0$		$\omega = 0.4$		$\omega = 0.7$		$\omega = 1$	
$\boldsymbol{\alpha}$	0.0334	0.0267	0.0342	0.0193	0.0550	0.0496	0.0647	0.0228
		0.0213		0.0836		0.0342		0.3761
		0.0412		0.0418		0.1218		0.1148
$\boldsymbol{\beta}$	0.0914	0.0917	0.0916	0.0896	0.1390	0.0558	0.4536	0.0117
		0.0095		0.0530		0.0232		0.0391
		0.0090		0.0078		0.1122		0.0103
$\boldsymbol{\zeta}$	4.8347	0.0209	5.0294	0.0219	5.0572	0.0531	5.2474	0.2284
		0.1582		0.0637		0.0364		0.2437
		0.0753		0.0545		0.0183		0.1892

Table 7:.Bayes estimates and Standard errors of the OGE-IW parameters for three real datasets under balanced squared error loss function based on Type II censoring under different samples ($\omega = 0.3$ and $r = 0.8n$).

Application	n	r	$\boldsymbol{\Psi}$	Estimates		SEs	
				$\breve{\boldsymbol{\Psi}}_B$	$\breve{\boldsymbol{\Psi}}_{EB}$	$\breve{\boldsymbol{\Psi}}_B$	$\breve{\boldsymbol{\Psi}}_{EB}$
I	53	42	α	0.7924	0.1874	0.0247	0.0285
					0.8266		0.0201
					1.0685		0.0354
			β	0.8650	0.5810	0.0515	0.0094
					0.5061		0.0098
					0.5339		0.0161
			ζ	1.0144	1.2258	0.0248	0.0227
					1.0453		0.0117
					1.2052		0.0140
II	30	24	α	0.9985	1.3193	0.0770	0.0298
					1.1522		0.0257
					1.2476		0.0275
			β	0.9225	0.8817	0.0697	0.0202
					0.7670		0.0154
					0.9390		0.0135
			ζ	0.9399	1.1668	0.0476	0.0260
					1.1045		0.0244
					0.6505		0.0179
III	60	48	α	1.2178	1.4519	0.0229	0.0185
					0.9048		0.0207
					0.9355		0.0265
			β	0.5256	0.3812	0.0210	0.0112
					0.1419		0.0275
					0.4729		0.0122
			ζ	0.9489	0.9510	0.0560	0.0106
					0.9353		0.0082
					0.9984		0.0201

Table 8. Bayes estimates and Standard errors of the OGEIW parameters for three real datasets under balanced linear exponential loss function based on Type II censoring under different samples ($\omega = 0.3, \nu = -1.8$ and $r = 0.8n$)

Application	n	r	$\underline{\boldsymbol{\Psi}}$	Estimates		SEs	
				$\underline{\breve{\boldsymbol{\Psi}}}_{\boldsymbol{B}}$	$\underline{\breve{\boldsymbol{\Psi}}}_{\boldsymbol{EB}}$	$\underline{\breve{\boldsymbol{\Psi}}}_{\boldsymbol{B}}$	$\underline{\breve{\boldsymbol{\Psi}}}_{\boldsymbol{EB}}$
I	53	42	$\boldsymbol{\alpha}$	0.6114	0.6533	0.0129	0.0035
					0.4887		0.0075
					0.5215		0.0058
			$\boldsymbol{\beta}$	0.3008	0.4363	0.0133	0.0087
					0.2380		0.0082
					0.2808		0.0044
			$\boldsymbol{\zeta}$	0.6521	0.6574	0.0174	0.0027
					0.6774		0.0043
					0.6626		0.0036
II	30	24	$\boldsymbol{\alpha}$	0.7345	0.6772	0.0218	0.0127
					0.7130		0.0047
					0.7079		0.0068
			$\boldsymbol{\beta}$	0.1274	0.0594	0.0267	0.0066
					0.1670		0.0065
					0.1401		0.0041
			$\boldsymbol{\zeta}$	0.7994	0.8294	0.0188	0.0050
					0.9091		0.0106
					0.8010		0.0049
III	60	48	$\boldsymbol{\alpha}$	0.7205	0.7408	0.0191	0.0035
					0.7104		0.0051
					0.7981		0.0105
			$\boldsymbol{\beta}$	0.3298	0.3037	0.0176	0.0034
					0.3805		0.0056
					0.2630		0.0031
			$\boldsymbol{\zeta}$	0.4714	0.4567	0.0270	0.0052
					0.4609		0.0023
					0.4618		0.0036

التقدير البييزى وتوقع البييزى للتوزيع الأسى العام المرجح بمعكوس وايبل

أسماء عبد العظيم محمد، رضا محمد حسن رفاعى، جنات رمضان الديان

كلية تجارة بنات ـ جامعة الازهر ـ القاهرة

الملخص: تناول هذا البحث إيجاد مقدرات بييز ومقدرات توقع بييز بنقطة وبفترة للتوزيع الأسى العام المرجح بمعكوس وايبل ، وذالك فى حالة كون جميع معالم التوزيع غير معلومة وتكون العينات مراقبة من النوع الثانى وبإفتراض أن التوزيع القبلى المشترك للمعالم معلم. وقد تم إستخدام التوزيعات الفائقة المنتظمة ثنائية المتغيرللحصول على مقدرات توقع بييز. كذالك تم إستخدام دوال خسارة متماثلة مثل دالة خسارة مربع الخطأ المتوازنة وغير متماثلة مثل دالة الخسارة الأسية الخطية المتوازنة كحالات خاصة لدالة الخسارة المتوازنة . فتم إستخدام R programming language لإيجاد التقديرات ولحساب المخاطرة ولحساب المخاطرة المقدرة . وتم إجراء مقارنات عددية لمختلف التقديرات الناتجة بإستخدام أسلوب المحاكاة وأيضا إستخدام ثلاث مجموعات من البيانات الحقيقية مع التطبيق على معدل وفيات كوفيد-19 لإيضاح الأهمية والكيفية التطبيقية للنتائج التى تم التوصل إليها.

الكلمات الإفتتاحية: التوزيع الأسى العام المرجح بمعكوس وايبل ، العينات مراقبة من النوع الثانى ، التقدير البييزى ،توقع البييزى ، وظيفة الخسارة المتوازنة.