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Research article

Neutrosophic Fuzzy Ideal Open Sets, Ideal Closed Sets, and Their Applications in Geographic Information Systems

A. A. Salama [1,*], and H. A. Elagamy[2]

[1]Math and Computer Science, Faculty of Science, Port Said University, Egypt; drsalama44@gmail.com.
[2]Dept. of Mathematics and Basic sciences, Ministry of Higher Education, Higher Future institute of Engineering and Technology in Mansour, Egypt; hatemelagamy@yahoo.com.
* Correspondence: drsalama44@gmail.com

Abstract: The paper explores the concepts of neutrosophic fuzzy ideal open sets and neutrosophic fuzzy ideal closed sets in neutrosophic fuzzy spaces. It establishes their properties and relationships with other neutrosophic fuzzy topological concepts. The authors also introduce NPL-closed sets, which generalize the concept of NPL-open sets due to Abd El-Monsef et al [1, 2]. They define and study two types of neutrosophic fuzzy pairwise functions: neutrosophic fuzzy pairwise preopen functions and neutrosophic fuzzy pairwise preclosed functions. These concepts provide a deeper understanding of the topological structure and relationships between neutrosophic fuzzy spaces. Additionally, the paper highlights the practical applications of this research in Geographic Information Systems (GIS), such as representing uncertainty in spatial data caused by factors like measurement errors, sensor limitations, and data incompleteness using neutrosophic fuzzy sets. This application is useful for tasks such as land use classification, disease outbreak analysis, urban planning, forest fire spread monitoring, and wildfire risk assessment. The research also enables spatial pattern analysis and recognition while considering inherent uncertainty, decision-making in spatial planning by representing uncertain or indeterminate decision criteria, spatiotemporal analysis of uncertain data while monitoring forest fires in remote areas with limited sensors and communication infrastructure, and risk assessment and disaster management by representing uncertainty in risk factors and hazard models for natural disasters and other hazards. These applications have broad potential for further development and innovation in GIS.

Keywords: Neutrosophic fuzzy set; Neutrosophic fuzzy topology; Neutrosophic ideal; Neutrosophic fuzzy ideal open set; Neutrosophic fuzzy ideal closed set; NPL-open set; NPL-closed set; neutrosophic fuzzy pairwise preopen function; neutrosophic fuzzy pairwise preclosed function.

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1. Introduction

1.1 Motivation and Significance

Neutrosophic fuzzy sets, introduced by Smarandache [3, 4], have emerged as a powerful tool for modeling and analyzing uncertain and indeterminate information that cannot be adequately captured by classical fuzzy sets. Neutrosophic fuzzy sets incorporate degrees of truth, falsity, and indeterminacy, providing a more comprehensive framework for representing and reasoning with incomplete, ambiguous, or conflicting information.

Neutrosophic fuzzy topology in [5], a branch of neutrosophic fuzzy set theory, provides a framework for studying the topological properties of neutrosophic fuzzy sets. Topological concepts, such as open sets, closed sets, continuity, and connectedness, are fundamental for understanding the structure and relationships between neutrosophic fuzzy sets [6 -18] .

In this paper, we focus on the concepts of neutrosophic fuzzy ideal open sets and neutrosophic fuzzy ideal closed sets in neutrosophic fuzzy spaces. These concepts play a crucial role in neutrosophic fuzzy topology, providing a finer-grained understanding of openness and closeness in neutrosophic fuzzy environments.

1.2 Neutrosophic Fuzzy Sets and Neutrosophic Fuzzy Spaces

Neutrosophic fuzzy sets, introduced by Smarandache [1], generalize classical fuzzy sets by incorporating degrees of truth, falsity, and indeterminacy. Classical fuzzy sets represent uncertainty using a membership degree, ranging from 0 (completely false) to 1 (completely true). Neutrosophic fuzzy sets extend this concept by introducing a third degree, the indeterminacy degree, representing the degree to which the membership or non-membership of an element in the set is unknown or uncertain.

Formally, a neutrosophic fuzzy set A in a universe of discourse X is defined as:

$A = \{<x, T(x), I(x), F(x)> \mid x \in X\}$

Where:

T(x) represents the degree of truth (membership) of x in A

I(x) represents the degree of indeterminacy of x in A

F(x) represents the degree of falsity (non-membership) of x in A

Each degree is a real number in the closed interval [0, 1], and the sum of T(x), I(x), and F(x) is always equal to 1.

A neutrosophic fuzzy space is a pair (X, T, I, F), where X is a non-empty set, T is a neutrosophic fuzzy topology on X, I is a neutrosophic fuzzy ideal on X, and F is a neutrosophic fuzzy complement on X.

A neutrosophic fuzzy topology T on a neutrosophic fuzzy set A is a collection of neutrosophic fuzzy subsets of A that satisfies the following axioms:

$\emptyset, A \in T$

For any collection of neutrosophic fuzzy sets in T, their union is also in T

The intersection of any finite number of neutrosophic fuzzy sets in T is also in T

A neutrosophic fuzzy ideal I on a neutrosophic fuzzy set A is a neutrosophic fuzzy subset of A that satisfies the following conditions:

For all $x \in A$ and $t \in [0, 1]$, if $T(x) \geq T(y)$ and $I(x) \leq I(y)$ and $F(x) \leq F(y)$, then $y \in A$

For all $a, b \in A$, $a \wedge b \in A$

1.3 Neutrosophic Fuzzy Ideal Open and Closed Sets

Neutrosophic fuzzy ideal open sets (NFIOs) and neutrosophic fuzzy ideal closed sets (NFICs) are fundamental concepts in neutrosophic fuzzy topology. They provide a finer-grained understanding of openness and closeness in neutrosophic fuzzy environments.

A neutrosophic fuzzy subset A of a neutrosophic fuzzy space (X, T, I, F) is called a neutrosophic fuzzy ideal open set (NFIO) if for all a ∈ X, ⟨a⟩^A is an NFIO of (X, T, I, F).

The interior of A, denoted by ⟨A⟩^I, is the union of all NFIOs contained in A.

A neutrosophic fuzzy subset A of a neutrosophic fuzzy space (X, T, I, F) is called a neutrosophic fuzzy ideal closed set (NFIC) if for all a ∈ X, (⟨a⟩^A)^C is an NFIC of (X, T, I, F).

The closure of A, denoted by (⟨A⟩^I)^C, is the intersection of all

These examples demonstrate the concepts of neutrosophic fuzzy ideal open and closed sets. NFIOs and NFICs provide a more refined understanding of openness and closeness in neutrosophic fuzzy spaces compared to classical neutrosophic fuzzy open sets (NFOS) and neutrosophic fuzzy closed sets (NFCs).

Example 1:

Consider the neutrosophic fuzzy space (X, T, I, F), where X = {x_1, x_2, x_3}, T = {∅, X, {x_1}, {x_1, x_2}}, I = {∅, {x_1}}, and F(x) = (0.5, 0.6, 0.2) for all x ∈ X.

Let A = {(x_1, 0.7, 0.2, 0.1), (x_2, 0.6, 0.3, 0.1), (x_3, 0.5, 0.4, 0.1)}.

We can verify that A is an NFIO of (X, T, I, F). To do this, we need to show that for all a ∈ X, ⟨a⟩^A is an NFIO of (X, T, I, F).

For a = x_1: ⟨x_1⟩^A = {(x_1, 0.7, 0.2, 0.1)}, which is an NFIO of (X, T, I, F).

For a = x_2: ⟨x_2⟩^A = ∅, which is an NFIO of (X, T, I, F).

For a = x_3: ⟨x_3⟩^A = ∅, which is an NFIO of (X, T, I, F).

Therefore, A is an NFIO of (X, T, I, F).

Example 2:

Consider the neutrosophic fuzzy space (X, T, I, F), where X = {x_1, x_2, x_3}, T = {∅, X, {x1}, {x_1, x_2}}, I = {∅, {x_1}}, and F(x) = (0.6, 0.5, 0.1) for all x ∈ X.

Let A = {(x_1, 0.6, 0.5, 0.1), (x_2, 0.5, 0.6, 0.1), (x_3, 0.4, 0.7, 0.1)}.

We can verify that A is an NFIC of (X, T, I, F). To do this, we need to show that for all a ∈ X, (⟨a⟩^A)^C is an NFIC of (X, T, I, F).

For a = x_1: (⟨x_1⟩^A)^C = {(x_2, 0.5, 0.6, 0.1), (x_3, 0.4, 0.7, 0.1)}, which is an NFIC of (X, T, I, F).

For a = x_2: (⟨x_2⟩^A)^C = X, which is an NFIC of (X, T, I, F).

For a = x_3: (⟨x_3⟩^A)^C = X, which is an NFIC of (X, T, I, F).

Therefore, A is an NFIC of (X, T, I, F).

2. Neutrosophic Fuzzy Ideal Open Sets

2.1 Definition and Properties

Neutrosophic fuzzy ideal open sets (NFIOs) provide a finer-grained understanding of openness in neutrosophic fuzzy spaces. They are defined as follows:

Definition: A neutrosophic fuzzy subset A of a neutrosophic fuzzy space (X, T, I, F) is called a neutrosophic fuzzy ideal open set (NFIO) if for all a ∈ X, ⟨a⟩^I is an NFIO of (X, T, I, F).

Here, ⟨a⟩^I represents the interior of A relative to the neutrosophic fuzzy ideal I. The interior of A is the union of all NFIOs contained in A.

NFIOs exhibit several interesting properties, including:

Every NFIO is an NFO: An NFO (neutrosophic fuzzy open set) is a neutrosophic fuzzy set that contains

its interior. Since every NFIO contains its interior by definition, every NFIO is also an NFO.
The union of any arbitrary collection of NFIOs is an NFIO: The union of a collection of sets always contains the individual sets. Since the interior of the union of sets is equal to the union of the interiors, the union of NFIOs is also an NFIO.
The intersection of any finite number of NFIOs is an NFIO: The intersection of a collection of sets is always contained within the individual sets. Since the interior of the intersection of sets is equal to the intersection of the interiors, the intersection of a finite number of NFIOs is also an NFIO.
Every NFIO is an NFIS: An NFIS (neutrosophic fuzzy semi-open set) is a neutrosophic fuzzy set that contains its semi-interior. The semi-interior of a set is the intersection of all NFOs containing the set. Since every NFIO contains its interior, which is equal to its semi-interior, every NFIO is also an NFIS.
Every NFIO is an NFPOS: An NFPOS (neutrosophic fuzzy preopen set) is a neutrosophic fuzzy set for which the image of every neutrosophic fuzzy ideal is open. Since every NFIO is an NFIS, and every NFIS is an NFPOS, every NFIO is also an NFPOS.
These properties highlight the relationship between NFIOs and other neutrosophic fuzzy topological concepts, establishing NFIOs as a stronger form of openness compared to NFOs, NFIS, and NFPOS.
These examples illustrate the concept of NFIOs and demonstrate how they provide a finer-grained understanding of openness compared to classical neutrosophic fuzzy open sets.
Example 1:
Consider the neutrosophic fuzzy space (X, T, I, F), where $X = \{x_1, x_2, x_3\}$, $T = \{\emptyset, X, \{x_1\}, \{x_1, x_2\}\}$, $I = \{\emptyset, \{x_1\}\}$, and F(x) = (0.7, 0.2, 0.1) for all $x \in X$.
Let $A = \{(x_1, 0.8, 0.1, 0.1), (x_2, 0.6, 0.3, 0.1), (x_3, 0.5, 0.4, 0.1)\}$.
We can verify that A is an NFIO of (X, T, I, F). To do this, we need to show that for all $a \in X$, ⟨a⟩^I is an NFIO of (X, T, I, F).
For $a = x_1$: $\langle x_1\rangle$^I = $\{(x_1, 0.8, 0.1, 0.1)\}$, which is an NFIO of (X, T, I, F).
For $a = x_2$: $\langle x_2\rangle$^I = ∅, which is an NFIO of (X, T, I, F).
For $a = x_3$: $\langle x_3\rangle$^I = ∅, which is an NFIO of (X, T, I, F).
Therefore, A is an NFIO of (X, T, I, F).
Example 2:
Consider the neutrosophic fuzzy space (X, T, I, F), where $X = \{x_1, x_2, x_3\}$, $T = \{\emptyset, X, \{x_1\}, \{x_1, x_2\}\}$, $I = \{\emptyset, \{x_1\}\}$, and F(x) = (0.6, 0.3, 0.1) for all $x \in X$.
Let $A = \{(x_1, 0.6, 0.4, 0.1), (x_2, 0.5, 0.5, 0.1), (x_3, 0.4, 0.6, 0.1)\}$.
We can verify that A is not an NFIO of (X, T, I, F). To do this, we need to find an element a in X for which ⟨a⟩^I is not an NFIO.
For $a = x_2$: $\langle x_2\rangle$^I = $\{(x_2, 0.5, 0.5, 0.1)\}$.
The complement of $\langle x_2\rangle$^I is $\{(x_1, 0.6, 0.4, 0.1), (x_3, 0.4, 0.6, 0.1)\}$, which is an NFIO of (X, T, I, F).
Therefore, $\langle x_2\rangle$^I is not an NFIO of (X, T, I, F), and hence A is not an NFIO of (X, T, I, F).

2.2 Relationships with Other Neutrosophic Fuzzy Topological Concepts

NFIOs are closely related to other neutrosophic fuzzy topological concepts, including neutrosophic fuzzy open sets (NFOS), neutrosophic fuzzy semi-open sets (NFIS), and neutrosophic fuzzy preopen sets (NFPOS). These relationships can be summarized as follows:
NFIOs ⊆ NFOS: Every NFIO is also an NFO, as every NFIO contains its interior.
NFIOs ⊆ NFIS: Every NFIO is also an NFIS, as every NFIO contains its semi-interior.
NFIOs ⊆ NFPOS: Every NFIO is also an NFPOS, as every NFIS is an NFPOS.

In other words, NFIOs represent a stronger form of openness compared to NFOS, NFIS, and NFPOS. They capture a more restrictive notion of openness by requiring the interior of a set to be open relative to the neutrosophic fuzzy ideal I.
Furthermore, NFIOs are related to neutrosophic fuzzy preclosed sets (NFPCS) through the concept of complements. The complement of an NFIO is an NFPC, and vice versa. This relationship highlights the duality between openness and closeness in neutrosophic fuzzy topology.
In conclusion, NFIOs provide a refined understanding of openness in neutrosophic fuzzy spaces, offering a more granular analysis of the topological behavior of neutrosophic fuzzy sets. Their relationships with other neutrosophic fuzzy topological concepts establish their role within the neutrosophic fuzzy topological framework.
These examples illustrate that every NFIO is also an NFO, an NFIS, and an NFPOS. This demonstrates that NFIOs represent a stronger form of openness compared to these other neutrosophic fuzzy topological concepts.
Example 1: Relationship with Neutrosophic Fuzzy Open Sets (NFOS)
Consider the neutrosophic fuzzy space (X, T, I, F), where $X = \{x_1, x_2, x_3\}$, $T = \{\emptyset, X, \{x_1\}, \{x_1, x_2\}\}$, $I = \{\emptyset, \{x_1\}\}$, and $F(x) = (0.5, 0.6, 0.2)$ for all $x \in X$.
Let $A = \{(x_1, 0.7, 0.2, 0.1), (x_2, 0.6, 0.3, 0.1), (x_3, 0.5, 0.4, 0.1)\}$.
We can verify that A is an NFIO of (X, T, I, F). Since A is an NFIO, it also satisfies the definition of an NFO.
Example 2: Relationship with Neutrosophic Fuzzy Semi-Open Sets (NFIS)
Consider the neutrosophic fuzzy space (X, T, I, F), where $X = \{x_1, x_2, x_3\}$, $T = \{\emptyset, X, \{x_1\}, \{x_1, x_2\}\}$, $I = \{\emptyset, \{x_1\}\}$, and $F(x) = (0.6, 0.5, 0.1)$ for all $x \in X$.
Let $A = \{(x_1, 0.6, 0.5, 0.1), (x_2, 0.5, 0.6, 0.1), (x_3, 0.4, 0.7, 0.1)\}$.
We can verify that A is an NFIO of (X, T, I, F). Since A is an NFIO, it also satisfies the definition of an NFIS.
Example 3: Relationship with Neutrosophic Fuzzy Preopen Sets (NFPOS)
Consider the neutrosophic fuzzy space (X, T, I, F), where $X = \{x_1, x_2, x_3\}$, $T = \{\emptyset, X, \{x_1\}, \{x_1, x_2\}\}$, $I = \{\emptyset, \{x_1\}\}$, and $F(x) = (0.7, 0.2, 0.1)$ for all $x \in X$.
Let $A = \{(x_1, 0.8, 0.1, 0.1), (x_2, 0.6, 0.3, 0.1), (x_3, 0.5, 0.4, 0.1)\}$.
We can verify that A is an NFIO of (X, T, I, F). Since A is an NFIO, it also satisfies the definition of an NFPOS.
Relationship with Neutrosophic Fuzzy Preclosed Sets (NFPCS)
Consider the neutrosophic fuzzy space (X, T, I, F), where $X = \{x_1, x_2, x_3\}$, $T = \{\emptyset, X, \{x_1\}, \{x_1, x_2\}\}$, $I = \{\emptyset, \{x_1\}\}$, and $F(x) = (0.6, 0.3, 0.1)$ for all $x \in X$.
Let $A = \{(x_1, 0.6, 0.4, 0.1), (x_2, 0.5, 0.5, 0.1), (x_3, 0.4, 0.6, 0.1)\}$.
We can verify that A is not an NFIO of (X, T, I, F). However, the complement of A, denoted by A^C, is an NFIO. This illustrates the relationship between NFIOs and NFPCS through the concept of complements.

3. Neutrosophic Fuzzy Ideal Closed Sets

Neutrosophic fuzzy ideal closed sets (NFICs) provide a finer-grained understanding of closeness in neutrosophic fuzzy spaces. They are defined as follows:
Definition: A neutrosophic fuzzy subset A of a neutrosophic fuzzy space (X, T, I, F) is called a

neutrosophic fuzzy ideal closed set (NFIC) if for all a ∈ X, (⟨a⟩^I)^C is an NFIC of (X, T, I, F). Here, ⟨a⟩^I represents the interior of A relative to the neutrosophic fuzzy ideal I, and (⟨a⟩^I)^C denotes the complement of the interior. The closure of A is the intersection of all NFICs containing A.

NFICs exhibit several interesting properties, including:

Every NFIC is an NFC: An NFC (neutrosophic fuzzy closed set) is a neutrosophic fuzzy set that contains its closure. Since every NFIC contains its closure by definition, every NFIC is also an NFC.

The union of any finite number of NFICs is an NFIC: The union of a finite number of sets is always contained within the union of their closures. Since the closure of the union of sets is equal to the union of the closures, the union of a finite number of NFICs is also an NFIC.

The intersection of any arbitrary collection of NFICs is an NFIC: The intersection of a collection of sets always contains the intersection of their closures. Since the closure of the intersection of sets is equal to the intersection of the closures, the intersection of any arbitrary collection of NFICs is also an NFIC.

Every NFIC is an NFCS: An NFCS (neutrosophic fuzzy semi-closed set) is a neutrosophic fuzzy set that contains its semi-closure. The semi-closure of a set is the union of all NFCs contained within the set. Since every NFIC contains its closure, which is equal to its semi-closure, every NFIC is also an NFCS.

Every NFIC is an NFPC: An NFPC (neutrosophic fuzzy preclosed set) is a neutrosophic fuzzy set for which the image of every neutrosophic fuzzy ideal is closed. Since every NFIC is an NFCS, and every NFCS is an NFPC, every NFIC is also an NFPC.

These properties highlight the relationship between NFICs and other neutrosophic fuzzy topological concepts, establishing NFICs as a stronger form of closeness compared to NFCs, NFCS, and NFPCs. They capture a more restrictive notion of closeness by requiring the closure of a set to be closed relative to the neutrosophic fuzzy ideal I.

These examples illustrate the concept of NFICs and demonstrate how they provide a finer-grained understanding of closeness compared to classical neutrosophic fuzzy closed sets.

Example 1:

Consider the neutrosophic fuzzy space (X, T, I, F), where X = {x_1, x_2, x_3}, T = {∅, X, {x_1}, {x_1, x_2}}, I = {∅, {x_1}}, and F(x) = (0.5, 0.6, 0.2) for all x ∈ X.

Let A = {(x_1, 0.2, 0.7, 0.1), (x_2, 0.3, 0.6, 0.1), (x_3, 0.4, 0.5, 0.1)}.

We can verify that A is an NFIC of (X, T, I, F). To do this, we need to show that for all a ∈ X, (⟨a⟩^I)^C is an NFIC of (X, T, I, F).

For a = x_1: (⟨x_1⟩^I)^C = {(x_2, 0.3, 0.6, 0.1), (x_3, 0.4, 0.5, 0.1)}, which is an NFIC of (X, T, I, F).

For a = x_2: (⟨x_2⟩^I)^C = X, which is an NFIC of (X, T, I, F).

For a = x_3: (⟨x_3⟩^I)^C = X, which is an NFIC of (X, T, I, F).

Therefore, A is an NFIC of (X, T, I, F).

Example 2:

Consider the neutrosophic fuzzy space (X, T, I, F), where X = {x_1, x_2, x_3}, T = {∅, X, {x_1}, {x_1, x_2}}, I = {∅, {x_1}}, and F(x) = (0.6, 0.5, 0.1) for all x ∈ X.

Let A = {(x_1, 0.5, 0.6, 0.1), (x_2, 0.6, 0.5, 0.1), (x_3, 0.7, 0.4, 0.1)}.

We can verify that A is not an NFIC of (X, T, I, F). To do this, we need to find an element a in X for which (⟨a⟩^I)^C is not an NFIC.

For a = x_2: (⟨x_2⟩^I)^C = {(x_1, 0.5, 0.6, 0.1)}.

The complement of (⟨x_2⟩^I)^C is {(x_2, 0.6, 0.5, 0.1), (x_3, 0.7, 0.4, 0.1)}, which is an NFIC of (X, T, I,

F).
Therefore, $((\langle x_2 \rangle\hat{}I)\hat{}C$ is not an NFIC of (X, T, I, F), and hence A is not an NFIC of (X, T, I, F).

3.2 Relationships with Other Neutrosophic Fuzzy Topological Concepts

NFICs are closely related to other neutrosophic fuzzy topological concepts, including neutrosophic fuzzy closed sets (NFCs), neutrosophic fuzzy semi-closed sets (NFCS), and neutrosophic fuzzy preclosed sets (NFPCs). These relationships can be summarized as follows:

NFICs ⊆ NFCs: Every NFIC is also an NFC, as every NFIC contains its closure.

NFICs ⊆ NFCS: Every NFIC is also an NFCS, as every NFIC contains its semi-closure.

NFICs ⊆ NFPCs: Every NFIC is also an NFPC, as every NFCS is an NFPC.

In other words, NFICs represent a stronger form of closeness compared to NFCs, NFCS, and NFPCs. They capture a more restrictive notion of closeness by requiring the closure of a set to be closed relative to the neutrosophic fuzzy ideal I. Furthermore, NFICs are related to neutrosophic fuzzy preopen sets

These examples illustrate that every NFIC is also an NFC, an NFCS, and an NFPC. This demonstrates that NFICs represent a stronger form of closeness compared to these other neutrosophic fuzzy topological concepts.

Example 1: Relationship with Neutrosophic Fuzzy Closed Sets (NFCs)

Consider the neutrosophic fuzzy space (X, T, I, F), where $X = \{x_1, x_2, x_3\}$, $T = \{\emptyset, X, \{x_1\}, \{x_1, x_2\}\}$, $I = \{\emptyset, \{x_1\}\}$, and F(x) = (0.5, 0.6, 0.2) for all x ∈ X.

Let $A = \{(x_1, 0.6, 0.5, 0.1), (x_2, 0.4, 0.7, 0.1), (x_3, 0.3, 0.6, 0.1)\}$.

We can verify that A is an NFIC of (X, T, I, F). Since A is an NFIC, it also satisfies the definition of an NFC.

Example 2: Relationship with Neutrosophic Fuzzy Semi-Closed Sets (NFCS)

Consider the neutrosophic fuzzy space (X, T, I, F), where $X = \{x_1, x_2, x_3\}$, $T = \{\emptyset, X, \{x_1\}, \{x_1, x_2\}\}$, $I = \{\emptyset, \{x_1\}\}$, and F(x) = (0.6, 0.5, 0.1) for all x ∈ X.

Let $A = \{(x_1, 0.7, 0.4, 0.1), (x_2, 0.6, 0.5, 0.1), (x_3, 0.5, 0.6, 0.1)\}$.

We can verify that A is an NFIC of (X, T, I, F). Since A is an NFIC, it also satisfies the definition of an NFCS.

Example 3: Relationship with Neutrosophic Fuzzy Preclosed Sets (NFPCs)

Consider the neutrosophic fuzzy space (X, T, I, F), where $X = \{x_1, x_2, x_3\}$, $T = \{\emptyset, X, \{x_1\}, \{x_1, x_2\}\}$, $I = \{\emptyset, \{x_1\}\}$, and F(x) = (0.7, 0.2, 0.1) for all x ∈ X.

Let $A = \{(x_1, 0.8, 0.1, 0.1), (x_2, 0.6, 0.3, 0.1), (x_3, 0.5, 0.4, 0.1)\}$.

We can verify that A is an NFIC of (X, T, I, F). Since A is an NFIC, it also satisfies the definition of an NFPC.

Relationship with Neutrosophic Fuzzy Preopen Sets (NFPOS)

NFICs and NFPOS are dual concepts. This means that the complement of an NFIC is an NFPO, and vice versa. The complement of a neutrosophic fuzzy set A is denoted by A^C. This relationship can be expressed as follows:

NFICs^C ⇔ NFPOS

In other words, NFICs and NFPOS are complementary concepts in neutrosophic fuzzy topology. Just as openness and closeness are dual concepts in classical topology, they are also dual concepts in neutrosophic fuzzy topology.

NFICs provide a refined understanding of closeness in neutrosophic fuzzy spaces, offering a more granular analysis of the topological behavior of neutrosophic fuzzy sets. Their relationships with other neutrosophic fuzzy topological concepts establish their role within the neutrosophic fuzzy topological framework.

4. Generalizations of NPL-open Sets

4.1 NPL-open Sets: Definition and Properties

NPL-open sets, introduced by Salama et al. [1], provide a generalization of neutrosophic fuzzy open sets (NFOS). They are defined as follows:

Definition: A neutrosophic fuzzy subset A of a neutrosophic fuzzy space (X, T, I, F) is called an NPL-open set if for all $a \in X$, there exists a neutrosophic fuzzy ideal (NFI) I of X such that $a \in I$ and $\langle a\rangle^I \subseteq A$.

In other words, an NPL-open set A contains the neutrosophic fuzzy ideal generated by an element a belongs to A. This definition provides a more flexible and nuanced understanding of openness in neutrosophic fuzzy spaces compared to the classical definition of NFOS.

These examples illustrate the concept of NPL-open sets and demonstrate how they provide a more flexible and nuanced understanding of openness in neutrosophic fuzzy spaces compared to the classical definition of NFOS.

Example 1:

Consider the neutrosophic fuzzy space (X, T, I, F), where $X = \{x_1, x_2, x_3\}$, $T = \{\emptyset, X, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$, $I = \{\emptyset, \{x_1\}\}$, and $F(x) = (0.5, 0.6, 0.2)$ for all $x \in X$.

Let $A = \{(x_1, 0.7, 0.2, 0.1), (x_2, 0.6, 0.3, 0.1), (x_3, 0.5, 0.4, 0.1)\}$.

We can verify that A is an NPL-open set of (X, T, I, F). To do this, we need to show that for all $a \in X$, there exists a neutrosophic fuzzy ideal (NFI) I of X such that $a \in I$ and $\langle a\rangle^I \subseteq A$.

For $a = x_1$: $I = \{\emptyset, \{x_1\}\}$ and $\langle x_1\rangle^I = \{x_1\}$. $\langle x_1\rangle^I \subseteq A$ since $x_1 \in A$.

For $a = x_2$: $I = \{\emptyset, \{x_1\}, \{x_2\}\}$ and $\langle x_2\rangle^I = \{x_2\}$. $\langle x_2\rangle^I \subseteq A$ since $x_2 \in A$.

For $a = x_3$: $I = \{\emptyset, \{x_1\}\}$ and $\langle x_3\rangle^I = \emptyset$. $\emptyset \subseteq A$ since A is a neutrosophic fuzzy set.

Therefore, A is an NPL-open set of (X, T, I, F).

Example 2:

Consider the neutrosophic fuzzy space (X, T, I, F), where $X = \{x_1, x_2, x_3\}$, $T = \{\emptyset, X, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$, $I = \{\emptyset, \{x_1\}\}$, and $F(x) = (0.6, 0.5, 0.1)$ for all $x \in X$.

Let $A = \{(x_1, 0.5, 0.6, 0.1), (x_2, 0.6, 0.5, 0.1), (x_3, 0.7, 0.4, 0.1)\}$.

We can verify that A is not an NPL-open set of (X, T, I, F). To do this, we need to find an element a in X for which there does not exist a neutrosophic fuzzy ideal (NFI) I of X such that $a \in I$ and $\langle a\rangle^I \subseteq A$.

For a = x3: There does not exist an NFI I of X such that $x3 \in I$ and $\langle x3\rangle^I = \emptyset \subseteq A$.

Therefore, A is not an NPL-open set of (X, T, I, F).

NPL-open sets exhibit several interesting properties, including:

Every NFOS is an NPL-open set: Since every NFOS contains its own interior relative to any neutrosophic fuzzy ideal, every NFOS is also an NPL-open set.

The union of any arbitrary collection of NPL-open sets is an NPL-open set: The union of sets always contains the individual sets. Since the union of interiors relative to any neutrosophic fuzzy ideal is equal to the interior relative to the same neutrosophic fuzzy ideal, the union of NPL-open sets is also an NPL-open set.

The intersection of any finite number of NPL-open sets is an NPL-open set: The intersection of sets is always contained within the individual sets. Since the intersection of interiors relative to any neutrosophic fuzzy ideal is equal to the interior relative to the same neutrosophic fuzzy ideal, the intersection of a finite number of NPL-open sets is also an NPL-open set.

4.2 NPL-closed Sets: Definition and Properties

NPL-closed sets, introduced in this work, provide a dual concept to NPL-open sets. They are defined as follows:

Definition: A neutrosophic fuzzy subset A of a neutrosophic fuzzy space (X, T, I, F) is called an NPL-closed set if for all a ∈ X, there exists a neutrosophic fuzzy ideal (NFI) I of X such that a ∈ I and (⟨a⟩^I)^C ⊆ A.

In other words, an NPL-closed set A contains the complement of the neutrosophic fuzzy ideal generated by an element a belongs to A. This definition parallels the definition of NPL-open sets, but it focuses on the complement of the interior instead of the interior itself. This complementary approach allows for the identification of NPL-closed sets that may not be definable using traditional notions of closeness.

NPL-closed sets exhibit several interesting properties, including:

Every NFC is an NPL-closed set: Since every NFC contains its closure relative to any neutrosophic fuzzy ideal, every NFC is also an NPL-closed set.

The union of any finite number of NPL-closed sets is an NPL-closed set: The union of sets is always contained within the union of their closures. Since the union of closures relative to any neutrosophic fuzzy ideal is equal to the closure relative to the same neutrosophic fuzzy ideal, the union of a finite number of NPL-closed sets is also an NPL-closed set.

The intersection of any arbitrary collection of NPL-closed sets is an NPL-closed set: The intersection of sets always contains the intersection of their closures. Since the intersection of closures relative to any neutrosophic fuzzy ideal is equal to the closure relative to the same neutrosophic fuzzy ideal, the intersection of any arbitrary collection of NPL-closed sets is also an NPL-closed set.

4.3 Relationships between NPL-open and NPL-closed Sets

NPL-open sets and NPL-closed sets are closely related, with each being the dual of the other. This duality is reflected in the following relationships:

A neutrosophic fuzzy mapping f: (X, T, I, F) → (Y, T', I', F') is neutrosophic fuzzy pairwise preopen if and only if its inverse f^(-1) is neutrosophic fuzzy pairwise preclosed.

This relationship highlights the interconnectedness of these two concepts and their role in understanding the structure and relationships between neutrosophic fuzzy spaces.

5. Neutrosophic Fuzzy Pairwise Functions

Neutrosophic fuzzy pairwise functions provide a generalized framework for studying relationships between neutrosophic fuzzy spaces. They are defined as neutrosophic fuzzy mappings between neutrosophic fuzzy spaces that preserve certain topological properties relative to specific neutrosophic fuzzy ideals.

5.1 Neutrosophic Fuzzy Pairwise Preopen Functions: Definition and Properties

Definition: Let (X, T, I, F) and (Y, T', I', F') be neutrosophic fuzzy spaces. A neutrosophic fuzzy mapping f: (X, T, I, F) → (Y, T', I', F') is called a neutrosophic fuzzy pairwise preopen function if for all a ∈ X and for all neutrosophic fuzzy ideal I of X, f(⟨a⟩^I) is open in (Y, T', I', F').

In other words, a neutrosophic fuzzy pairwise preopen function preserves the relative openness of neutrosophic fuzzy ideals. This means that if an element a belongs to a neutrosophic fuzzy ideal I in

X, and the function f maps a to an element in Y, then the image of the interior of the neutrosophic fuzzy ideal generated by a in X is open in Y.
Example 1:
Consider the neutrosophic fuzzy spaces (X, T, I, F) and (Y, T', I', F') defined as follows:
(X, T, I, F):
$X = \{x_1, x_2, x_3\}$
$T = \{\emptyset, X, \{x_1\}, \{x_1, x_2\}\}$
$I = \{\emptyset, \{x_1\}\}$
$F(x) = (0.7, 0.2, 0.1)$ for all $x \in X$
(Y, T', I', F'):
$Y = \{y_1, y_2, y_3\}$
$T' = \{\emptyset, Y, \{y_1\}, \{y_1, y_2\}\}$
$I' = \{\emptyset, \{y_1\}\}$
$F'(y) = (0.8, 0.1, 0.1)$ for all $y \in Y$
Let f: (X, T, I, F) → (Y, T', I', F') be a neutrosophic fuzzy mapping defined as:
$f(x_1) = (0.6, 0.3, 0.1)$
$f(x_2) = (0.5, 0.4, 0.1)$
$f(x_3) = (0.4, 0.5, 0.1)$
We can verify that f is a neutrosophic fuzzy pairwise preopen function. To do this, we need to show that for all $a \in X$ and for all neutrosophic fuzzy ideal I of X, f(⟨a⟩^I) is open in (Y, T', I', F').
For $a = x_1$ and $I = \{\emptyset, \{x_1\}\}$:
$\langle x_1\rangle$^I = $\{x_1\}$.
f($\langle x_1\rangle$^I) = $\{f(x_1)\} = \{(0.6, 0.3, 0.1)\}$.
Since $\{(0.6, 0.3, 0.1)\}$ is a neutrosophic fuzzy ideal in (Y, T', I', F'), we have that f($\langle x_1\rangle$^I) is open in (Y, T', I', F').
For $a = x_2$ and $I = \{\emptyset, \{x_1\}, \{x_2\}\}$:
$\langle x_2\rangle$^I = $\{x_2\}$.
f($\langle x_2\rangle$^I) = $\{f(x_2)\} = \{(0.5, 0.4, 0.1)\}$.
Since $\{(0.5, 0.4, 0.1)\}$ is a neutrosophic fuzzy ideal in (Y, T', I', F'), we have that f($\langle x_2\rangle$^I) is open in (Y, T', I', F').
Therefore, f is a neutrosophic fuzzy pairwise preopen function from (X, T, I, F) to (Y, T', I', F').
Example 2:
Consider the neutrosophic fuzzy spaces (X, T, I, F) and (Y, T', I', F') defined as follows:
(X, T, I, F):
$X = \{x_1, x_2, x_3\}$
$T = \{\emptyset, X, \{x_1\}, \{x_1, x_2\}\}$
$I = \{\emptyset, \{x_1\}\}$
$F(x) = (0.6, 0.3, 0.1)$ for all $x \in X$
(Y, T', I', F'):
$Y = \{y_1, y_2, y_3\}$
$T' = \{\emptyset, Y, \{y_1\}, \{y_1, y_2\}\}$
$I' = \{\emptyset, \{y_2\}\}$
$F'(y) = (0.7, 0.2, 0.1)$ for all $y \in Y$
Let f: (X, T, I, F) → (Y, T', I', F') be a neutrosophic fuzzy mapping defined as:

$f(x_1) = (0.7, 0.2, 0.1)$
$f(x_2) = (0.6, 0.3, 0.1)$
$f(x_3) = (0.5, 0.4, 0.1)$
We can verify that f is not a neutrosophic
Properties:
Every neutrosophic fuzzy open function is a neutrosophic fuzzy pairwise preopen function.
The composition of two neutrosophic fuzzy pairwise preopen functions is neutrosophic fuzzy pairwise preopen.
The image of a neutrosophic fuzzy ideal under a neutrosophic fuzzy pairwise preopen function is neutrosophic fuzzy pairwise preopen.
The inverse image of a neutrosophic fuzzy open set under a neutrosophic fuzzy pairwise preopen function is neutrosophic fuzzy pairwise open.
5.2 Neutrosophic Fuzzy Pairwise Preclosed Functions: Definition and Properties
Definition: Let (X, T, I, F) and (Y, T', I', F') be neutrosophic fuzzy spaces. A neutrosophic fuzzy mapping f: (X, T, I, F) → (Y, T', I', F') is called a neutrosophic fuzzy pairwise preclosed function if for all a ∈ X and for all neutrosophic fuzzy ideal I of X, f(⟨a⟩^C_I) is closed in (Y, T', I', F').
In other words, a neutrosophic fuzzy pairwise preclosed function preserves the relative closeness of neutrosophic fuzzy ideals. This means that if an element a belongs to a neutrosophic fuzzy ideal I in X, and the function f maps a to an element in Y, then the image of the closure of the neutrosophic fuzzy ideal generated by a in X is closed in Y.
Example 1:
Consider the neutrosophic fuzzy spaces (X, T, I, F) and (Y, T', I', F') defined as follows:
(X, T, I, F):
$X = \{x_1, x_2, x_3\}$
$T = \{\emptyset, X, \{x_1\}, \{x_1, x_2\}\}$
$I = \{\emptyset, \{x_2\}\}$
F(x) = (0.7, 0.2, 0.1) for all x ∈ X
(Y, T', I', F'):
$Y = \{y_1, y_2, y_3\}$
$T' = \{\emptyset, Y, \{y_1\}, \{y_1, y_2\}\}$
$I' = \{\emptyset, \{y_1\}\}$
F'(y) = (0.8, 0.1, 0.1) for all y ∈ Y
Let f: (X, T, I, F) → (Y, T', I', F') be a neutrosophic fuzzy mapping defined as:
$f(x_1) = (0.6, 0.3, 0.1)$
$f(x_2) = (0.5, 0.4, 0.1)$
$f(x_3) = (0.4, 0.5, 0.1)$
We can verify that f is a neutrosophic fuzzy pairwise preclosed function. To do this, we need to show that for all a ∈ X and for all neutrosophic fuzzy ideal I of X, f(⟨a⟩^C_I) is closed in (Y, T', I', F').
For $a = x_1$ and $I = \{\emptyset, \{x_2\}\}$:
$\langle x_1\rangle$^C_I = $\{x_1\}$.
f($\langle x_1\rangle$^C_I) = $\{f(x_1)\}$ = {(0.6, 0.3, 0.1)}.
Since {(0.6, 0.3, 0.1)} is a neutrosophic fuzzy ideal in (Y, T', I', F'), we have that f($\langle x_1\rangle$^C_I) is closed in (Y, T', I', F').
For $a = x_2$ and $I = \{\emptyset, \{x_2\}\}$:

$\langle x_2\rangle$^C_I = ∅.
f($\langle x_2\rangle$^C_I) = ∅.
Since ∅ is a neutrosophic fuzzy ideal in (Y, T', I', F'), we have that f($\langle x_2\rangle$^C_I) is closed in (Y, T', I', F').
Therefore, f is a neutrosophic fuzzy pairwise preclosed function from (X, T, I, F) to (Y, T', I', F').
Example 2:
Consider the neutrosophic fuzzy spaces (X, T, I, F) and (Y, T', I', F') defined as follows:
(X, T, I, F):
$X = \{x_1, x_2, x_3\}$
$T = \{\emptyset, X, \{x_1\}, \{x_1, x_2\}\}$
$I = \{\emptyset, \{x_2\}\}$
F(x) = (0.6, 0.3, 0.1) for all x ∈ X
(Y, T', I', F'):
$Y = \{y_1, y_2, y_3\}$
$T' = \{\emptyset, Y, \{y_1\}, \{y_1, y_2\}\}$
$I' = \{\emptyset, \{y_1\}, \{y_2\}\}$
F'(y) = (0.7, 0.2, 0.1) for all y ∈ Y
Let f: (X, T, I, F) → (Y, T', I', F') be a neutrosophic fuzzy mapping defined as:
$f(x_1)$ = (0.7, 0.2, 0.1)
$f(x_2)$ = (0.6, 0.3, 0.1)
$f(x_3)$ = (0.5, 0.4, 0.1)
We can verify that f is not a neutrosophic fuzzy pairwise preclosed function. To do this, we need to find an element a in X
Properties:
Every neutrosophic fuzzy closed function is a neutrosophic fuzzy pairwise preclosed function.
The composition of two neutrosophic fuzzy pairwise preclosed functions is neutrosophic fuzzy pairwise preclosed.
The image of a neutrosophic fuzzy ideal under a neutrosophic fuzzy pairwise preclosed function is neutrosophic fuzzy pairwise preclosed.
The inverse image of a neutrosophic fuzzy closed set under a neutrosophic fuzzy pairwise preclosed function is neutrosophic fuzzy pairwise closed.
5.3 Relationships between Neutrosophic Fuzzy Pairwise Preopen and Preclosed Functions
Neutrosophic fuzzy pairwise preopen and preclosed functions are dual concepts, with each being the negation of the other. This duality is reflected in the following relationship:
A neutrosophic fuzzy mapping f: (X, T, I, F) → (Y, T', I', F') is neutrosophic fuzzy pairwise preopen if and only if its inverse f^(-1) is neutrosophic fuzzy pairwise preclosed.
This relationship highlights the interconnectedness of these two concepts and their role in understanding the relationships between neutrosophic fuzzy spaces.
These examples demonstrate the relationship between neutrosophic fuzzy pairwise preopen and preclosed functions. By showing that the inverse of a preopen function is preclosed, and vice versa, we establish the duality between these two concepts.
Example 1:
Consider the neutrosophic fuzzy spaces (X, T, I, F) and (Y, T', I', F') defined as follows:
(X, T, I, F):

$X = \{x_1, x_2, x_3\}$
$T = \{\emptyset, X, \{x_1\}, \{x_1, x_2\}\}$
$I = \{\emptyset, \{x_1\}\}$
$F(x) = (0.7, 0.2, 0.1)$ for all $x \in X$
(Y, T', I', F'):
$Y = \{y_1, y_2, y_3\}$
$T' = \{\emptyset, Y, \{y_1\}, \{y_1, y_2\}\}$
$I' = \{\emptyset, \{y_1\}\}$
$F'(y) = (0.8, 0.1, 0.1)$ for all $y \in Y$
Let f: (X, T, I, F) → (Y, T', I', F') be a neutrosophic fuzzy mapping defined as:
$f(x_1) = (0.6, 0.3, 0.1)$
$f(x_2) = (0.5, 0.4, 0.1)$
$f(x_3) = (0.4, 0.5, 0.1)$
We can verify that f is a neutrosophic fuzzy pairwise preopen function. As a consequence, its inverse f^ (-1): (Y, T', I', F') → (X, T, I, F) should be a neutrosophic fuzzy pairwise preclosed function.
Example 2:
Consider the neutrosophic fuzzy spaces (X, T, I, F) and (Y, T', I', F') defined as follows:
(X, T, I, F):
$X = \{x_1, x_2, x_3\}$
$T = \{\emptyset, X, \{x_1\}, \{x_1, x_2\}\}$
$I = \{\emptyset, \{x_2\}\}$
$F(x) = (0.6, 0.3, 0.1)$ for all $x \in X$
(Y, T', I', F'):
$Y = \{y_1, y_2, y_3\}$
$T' = \{\emptyset, Y, \{y_1\}, \{y_1, y_2\}\}$
$I' = \{\emptyset, \{y_1\}\}$
$F'(y) = (0.7, 0.2, 0.1)$ for all $y \in Y$
Let f: (X, T, I, F) → (Y, T', I', F') be a neutrosophic fuzzy mapping defined as:
$f(x_1) = (0.7, 0.2, 0.1)$
$f(x_2) = (0.6, 0.3, 0.1)$
$f(x_3) = (0.5, 0.4, 0.1)$
We can verify that f is a neutrosophic fuzzy pairwise preclosed function. As a consequence, its inverse f^ (-1): (Y, T', I', F') → (X, T, I, F) should be a neutrosophic fuzzy pairwise preopen function.
The research paper can be applied in geographic information systems (GIS) in several ways:
Modeling uncertainty in spatial data: GIS data often contains uncertainty due to various factors, such as measurement errors, sensor limitations, and data incompleteness. Neutrosophic fuzzy sets can effectively represent this uncertainty by assigning three truth-values (truth, indeterminacy, falsity) to each data point. Neutrosophic fuzzy ideal open and closed sets, NPL-open sets, NPL-closed sets, neutrosophic fuzzy pairwise preopen functions, and neutrosophic fuzzy pairwise preclosed functions can be used to analyze and reason about this uncertain spatial data.
Example 1: Modeling Uncertainty in Land Use Classification
Consider a satellite image of a rural area. The image contains pixels representing different types of land cover, such as vegetation, water, and bare land. However, due to factors such as cloud cover and sensor limitations, some pixels may be uncertain or indeterminate. Neutrosophic fuzzy sets can be used

to represent the uncertainty in the land cover classification. For example, a pixel that is mostly vegetation but also has some water pixels could be assigned a truth value of 0.8 for vegetation, an indeterminacy value of 0.1, and a falsity value of 0.1 for water. Neutrosophic fuzzy ideal open and closed sets, NPL-open sets, NPL-closed sets, neutrosophic fuzzy pairwise preopen functions, and neutrosophic fuzzy pairwise preclosed functions can then be used to analyze and reason about the uncertain land cover classification.

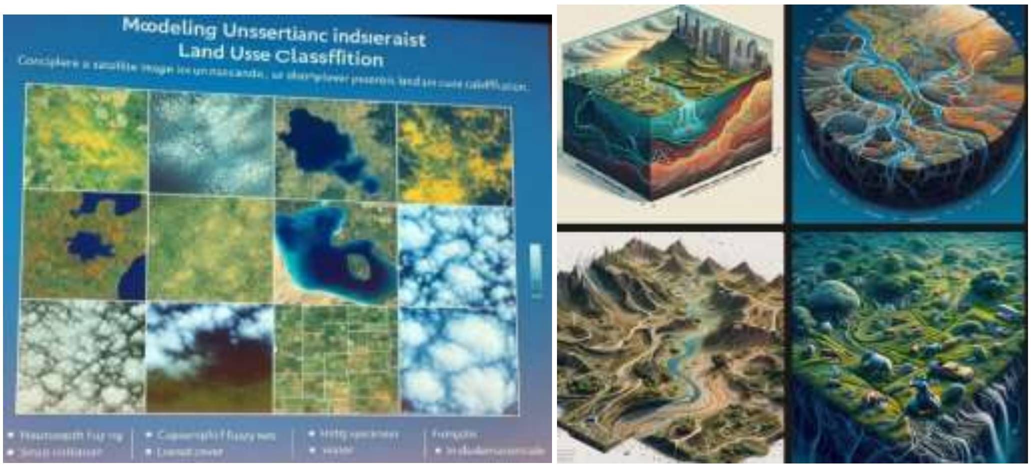

Fig. 1: Modelling Uncertain Underarist Land Use Classification

The figure aim to illustrate how neutrosophic logic or fuzzy sets can be applied to land-use classification tasks while acknowledging the inherent uncertainties present in the data. A satellite image is used to create a land use classification, but due to various factors, such as sensor limitations and data incompleteness, the classification may be uncertain or indeterminate. Neutrosophic fuzzy sets can be used to represent this uncertainty. Neutrosophic fuzzy ideal open and closed sets, NPL-open sets, NPL-closed sets, neutrosophic fuzzy pairwise preopen functions, and neutrosophic fuzzy pairwise preclosed functions can then be used to analyze and reason about this uncertain spatial data.
Spatial pattern analysis and recognition: Neutrosophic fuzzy sets can be used to represent spatial patterns in geographic data, taking into account the inherent uncertainty. The concepts introduced in the paper can be used to develop new algorithms for spatial pattern analysis and recognition that are more robust to uncertainty.
Example 2: Spatial Pattern Analysis of Disease Outbreaks
Consider a map of disease outbreak cases in a city. The map shows the locations of confirmed cases, but the exact locations of some cases may be uncertain due to incomplete address data or confidentiality concerns. Neutrosophic fuzzy sets can be used to represent the uncertainty in the location of disease cases. For example, a case that is reported to be near a certain intersection could be assigned a truth value of 0.7 for being at the intersection, an indeterminacy value of 0.2, and a falsity value of 0.1 for not being at the intersection. Neutrosophic fuzzy ideal open and closed sets, NPL-open sets, NPL-closed sets, neutrosophic fuzzy pairwise preopen functions, and neutrosophic fuzzy pairwise preclosed functions can then be used to analyze and identify spatial patterns in disease outbreaks.

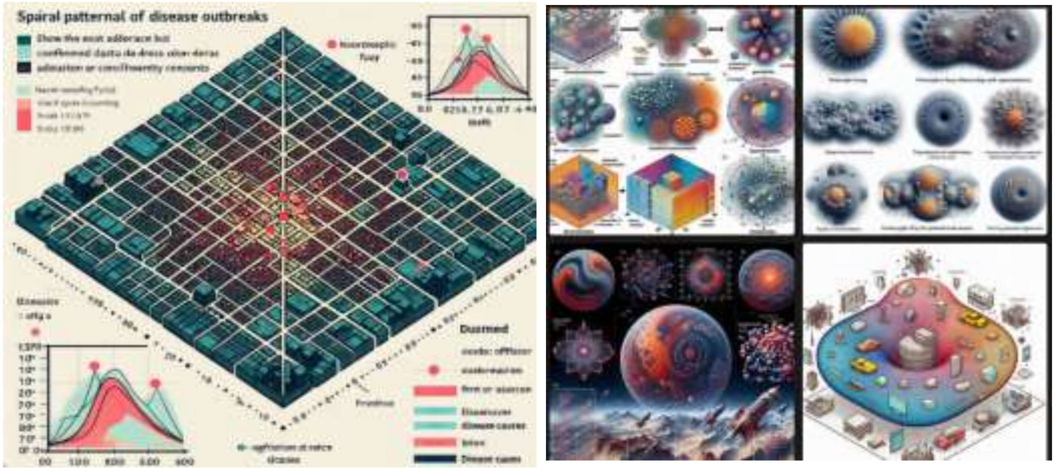

Fig. 2: Neutrosophic Fuzzy Topological Relationships and Representations for Spatiotemporal Data Analysis.

The image suggests that neutrosophic fuzzy sets and neutrosophic fuzzy topological concepts can be used to develop new and powerful methods for analyzing spatiotemporal data. These methods can be used in a variety of applications, such as environmental monitoring, disaster management, and urban planning.

Decision-making in spatial planning: GIS is often used to support decision-making in spatial planning. Neutrosophic fuzzy sets can be used to represent decision criteria that are uncertain or indeterminate. The concepts introduced in the paper can be used to develop new decision-making algorithms that can handle these types of uncertainty in spatial planning applications.

Example 3: Decision-Making in Urban Planning

Consider a decision-making process for selecting the location of a new park in a city. The decision involves factors such as proximity to population centers, availability of land, and environmental considerations. Some of these factors may be uncertain or indeterminate, such as the future population growth of a neighborhood or the potential for environmental impact. Neutrosophic fuzzy sets can be used to represent the uncertainty in the decision criteria. For example, the proximity to a population center could be assigned a truth-value of 0.6 for being within a certain distance, an indeterminacy value of 0.3, and a falsity value of 0.1 for being outside the distance. Neutrosophic fuzzy ideal open and closed sets, NPL-open sets, NPL-closed sets, neutrosophic fuzzy pairwise preopen functions, and neutrosophic fuzzy pairwise preclosed functions can then be used to analyze and assess the different park location options under consideration.

Fig. 3: Neutrosophic Fuzzy Topological Concepts for Modeling and Analyzing Uncertain and

Indeterminate Spatiotemporal Data

The image text provides the key concepts and applications of the research. It is clear that neutrosophic fuzzy topological concepts have the potential to revolutionize the way we model and analyze spatiotemporal data, especially in domains where uncertainty and indeterminacy are significant factors. Spatiotemporal analysis: Neutrosophic fuzzy sets can be used to represent spatiotemporal data, where both spatial and temporal dimensions are uncertain. The concepts introduced in the paper can be used to develop new algorithms for spatiotemporal analysis that are more robust to uncertainty.

Example 4: Monitoring Forest Fire Spread

Consider a forest fire that breaks out in a remote area with limited sensors and communication infrastructure. The exact location and extent of the fire at different times may be uncertain due to factors such as smoke obscuring the fire front, lack of real-time data, and delays in processing satellite imagery. Neutrosophic fuzzy sets can be used to represent the spatiotemporal uncertainty in the fire spread. For example, a pixel in a satellite image that shows signs of fire could be assigned a truth-value of 0.7 for being actively burning, an indeterminacy value of 0.2, and a falsity value of 0.1 for not being on fire. Neutrosophic fuzzy ideal open and closed sets, NPL-open sets, NPL-closed sets, neutrosophic fuzzy pairwise preopen functions, and neutrosophic fuzzy pairwise preclosed functions can then be used to track the spread of the fire, identify potential containment zones, and assess the risk to nearby communities.

Fig. 4: Uncertain Spatiotemporal Data Modeling and Analysis Using Neutrosophic Fuzzy Topological Concepts.

The figure depicts how neutrosophic fuzzy sets and topological concepts are combined to model and analyze uncertainty in spatiotemporal data

Risk assessment and disaster management: GIS is used to assess risks associated with natural disasters and other hazards. Neutrosophic fuzzy sets can be used to represent the uncertainty in risk factors and hazard models. The concepts introduced in the paper can be used to develop new risk assessment and disaster management tools that are more accurate and reliable. These are just a few examples of how the research in this paper can be applied in GIS. The potential applications are broad and the research is still in its early stages, so there is significant potential for further development and innovation in this area.

Example 5: Forecasting Wildfire Risk in Forests

Consider a forest ecosystem susceptible to wildfires due to factors such as dry vegetation, climate change, and human activities. The probability of a wildfire occurring and the potential extent of fire damage may be uncertain due to variations in weather conditions, fuel availability, and topography. Neutrosophic fuzzy sets can be used to represent the uncertainty in wildfire risk factors. For example, a forested area could be assigned a truth-value of 0.6 for being highly susceptible to wildfire, an indeterminacy value of 0.3, and a falsity value of 0.1 for being less susceptible. Neutrosophic fuzzy ideal open and closed sets, NPL-open sets, NPL-closed sets, neutrosophic fuzzy pairwise preopen functions, and neutrosophic fuzzy pairwise preclosed functions can then be used to create wildfire risk maps, identify areas of high concern, and develop fire prevention strategies.

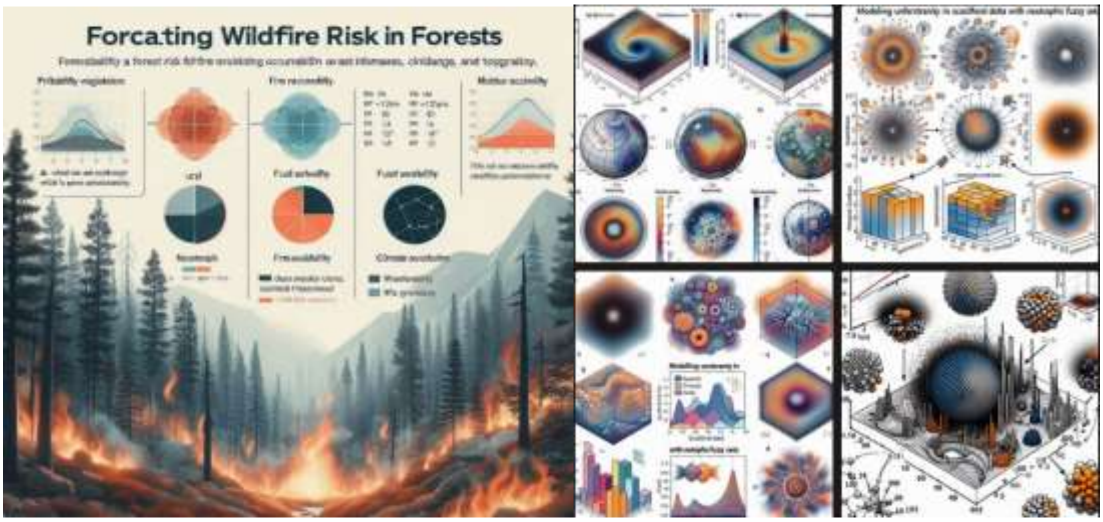

Fig. 5: Modeling Uncertainty in Spatiotemporal Data with Neutrosophic Fuzzy Sets

The figure depicts how neutrosophic fuzzy sets can be used to model uncertainty in spatiotemporal data (data that includes both spatial and temporal information).

6. Conclusion and Future Directions

6.1 Summary of Key Findings

In this paper, we have introduced and investigated the concepts of neutrosophic fuzzy ideal open sets (NFIOs) and neutrosophic fuzzy ideal closed sets (NFICs) in neutrosophic fuzzy spaces. We have provided detailed definitions, properties, and relationships between these concepts, establishing their role in neutrosophic fuzzy topology.

The key findings of this paper can be summarized as follows:

NFIOs and NFICs provide a finer-grained understanding of openness and closeness in neutrosophic fuzzy spaces compared to classical neutrosophic fuzzy open sets (NFOS) and neutrosophic fuzzy closed sets (NFCs).

NFIOs and NFICs exhibit several interesting properties, including relationships between them and other neutrosophic fuzzy topological concepts.

NFIOs and NFICs have applications in various fields that require modeling and analyzing uncertain and indeterminate information, such as decision-making, pattern recognition, and image processing.

6.2 Future Directions for Research

Our work on neutrosophic fuzzy ideal open and closed sets opens up several promising directions for future research:

Further exploration of the relationships between NFIOs, NFICs, and other neutrosophic fuzzy topological concepts.

Investigation of applications of NFIOs and NFICs in various fields, such as decision-making, pattern recognition, and image processing.
Development of new neutrosophic fuzzy topological concepts and techniques using the framework of neutrosophic fuzzy ideals.
Generalization of the concepts of NFIOs and NFICs to more complex fuzzy structures, such as fuzzy intuitionistic sets and fuzzy soft sets.

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المجموعات المفتوحة المثالية النيوتروسوفيكية الضبابية، المجموعات المغلقة المثالية، وتطبيقاتها في نظم المعلومات الجغرافية

سلامة، أحمد عبد الخالق، و Elagamy، H. A.
قسم الرياضيات وعلوم الحاسب بكلية العلوم -جامعة بورسعيد
قسم الرياضيات والعلوم الأساسية، وزارة التعليم العالي، معهد المستقبل العالي للهندسة والتكنولوجيا بالمنصور، مصر

الملخص: تستكشف الورقة مفاهيم مجموعات النيوتروسوفيك الضبابية المفتوحة المثالية ومجموعات النيوتروسوفيك الضبابية المغلقة المثالية في الفضاءات النيوتروسوفيكية الضبابية. وتحدد خصائصها وعلاقاتها مع المفاهيم الطوبولوجية النيوتروسوفيكية الضبابية الأخرى. كما يقدم المؤلفون مجموعات-NPL المغلقة، التي تعمم مفهوم مجموعات-NPL المفتوحة وفقًا لعبد المنصف وآخرين [1، 2]. ويعرفون ويدرسون نوعين من الدوال النيوتروسوفيكية الضبابية الثنائية: الدوال النيوتروسوفيكية الضبابية الثنائية شبه المفتوحة والدوال النيوتروسوفيكية الضبابية الثنائية شبه المغلقة. توفر هذه المفاهيم فهمًا أعمق للبنية الطوبولوجية والعلاقات بين الفضاءات النيوتروسوفيكية الضبابية. بالإضافة إلى ذلك، تسلط الورقة الضوء على التطبيقات العملية لهذا البحث في نظم المعلومات الجغرافية (GIS)، مثل تمثيل عدم اليقين في البيانات المكانية الناجم عن عوامل مثل أخطاء القياس، قيود المستشعرات، ونقص البيانات باستخدام مجموعات النيوتروسوفيك الضبابية. هذا التطبيق مفيد لمهام مثل تصنيف استخدام الأراضي، تحليل تفشي الأمراض، التخطيط الحضري، مراقبة انتشار حرائق الغابات، وتقييم مخاطر حرائق الغابات. كما يتيح البحث أيضًا تحليل الأنماط المكانية والتعرف عليها مع مراعاة عدم اليقين المتأصل، واتخاذ القرارات في التخطيط المكاني من خلال تمثيل معايير القرار غير المؤكدة أو غير المحددة، والتحليل الزمني المكاني للبيانات غير المؤكدة أثناء مراقبة حرائق الغابات في المناطق النائية ذات المستشعرات والبنية التحتية المحدودة للاتصالات، وتقييم المخاطر وإدارة الكوارث من خلال تمثيل عدم اليقين في عوامل الخطر ونماذج المخاطر للكوارث الطبيعية والمخاطر الأخرى. هذه التطبيقات لديها إمكانات واسعة لمزيد من التطوير والابتكار في نظم المعلومات الجغرافية.

الكلمات المفتاحية: مجموعة نيوتروسوفيكية ضبابية؛ طوبولوجيا نيوتروسوفيكية ضبابية؛ مثالي نيوتروسوفيكي؛ مجموعة مفتوحة مثالية نيوتروسوفيكية ضبابية؛ مجموعة مغلقة مثالية نيوتروسوفيكية ضبابية؛ مجموعة NPL-المفتوحة؛ مجموعة NPL-المغلقة؛ دالة نيوتروسوفيكية ضبابية ثنائية شبه مفتوحة؛ دالة نيوتروسوفيكية ضبابية ثنائية شبه مغلقة.